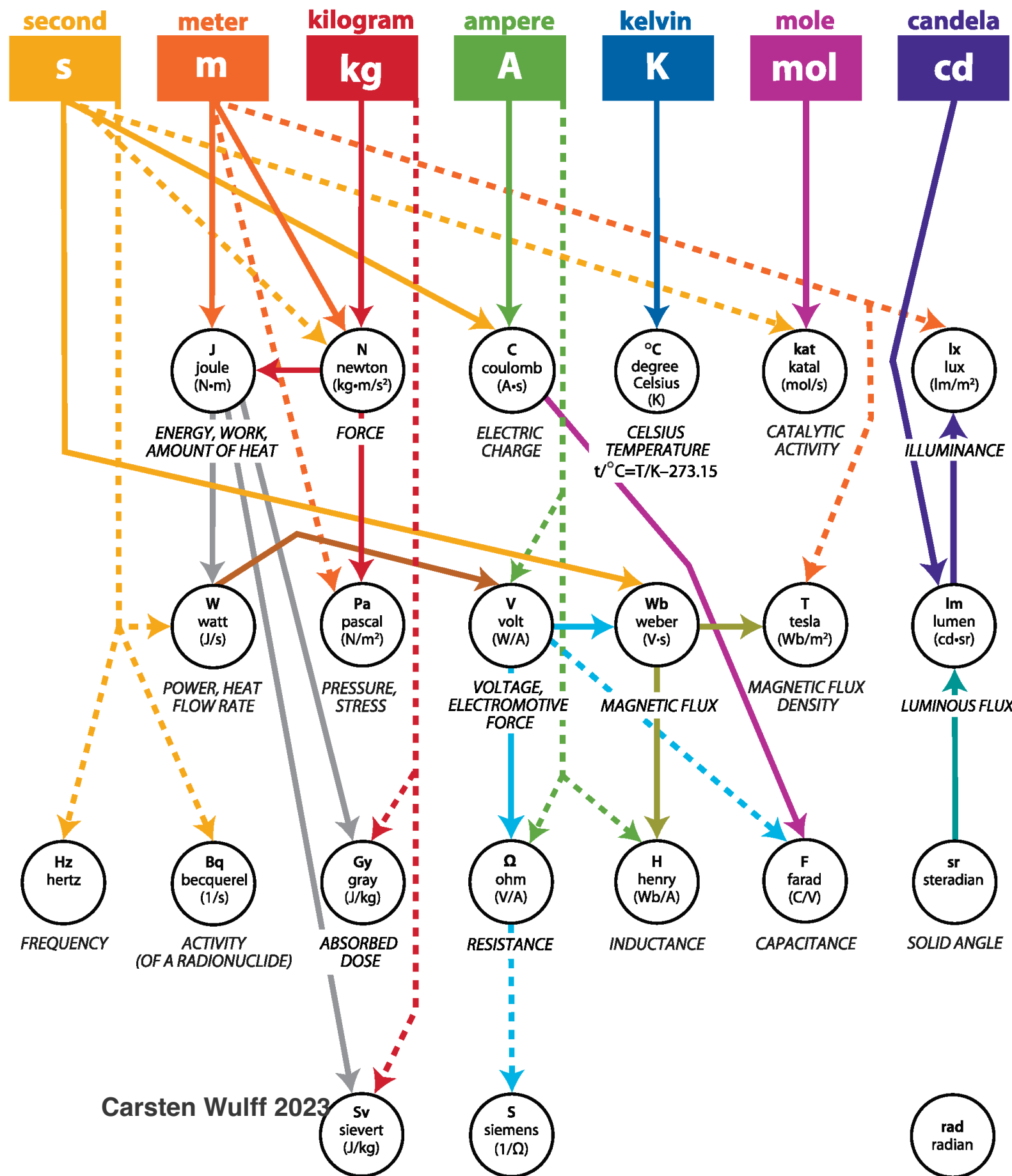date: 2023-10-26

A Refresher

There are standard units of measurement

All known physical quantities are derived from 7 base units (SI units)

- second (s) : time

- meter (m) : space

- kg (kilogram) : weight

- ampere (A) : current

- kelvin (K) : temperature

- candela (cd) : luminous intensity

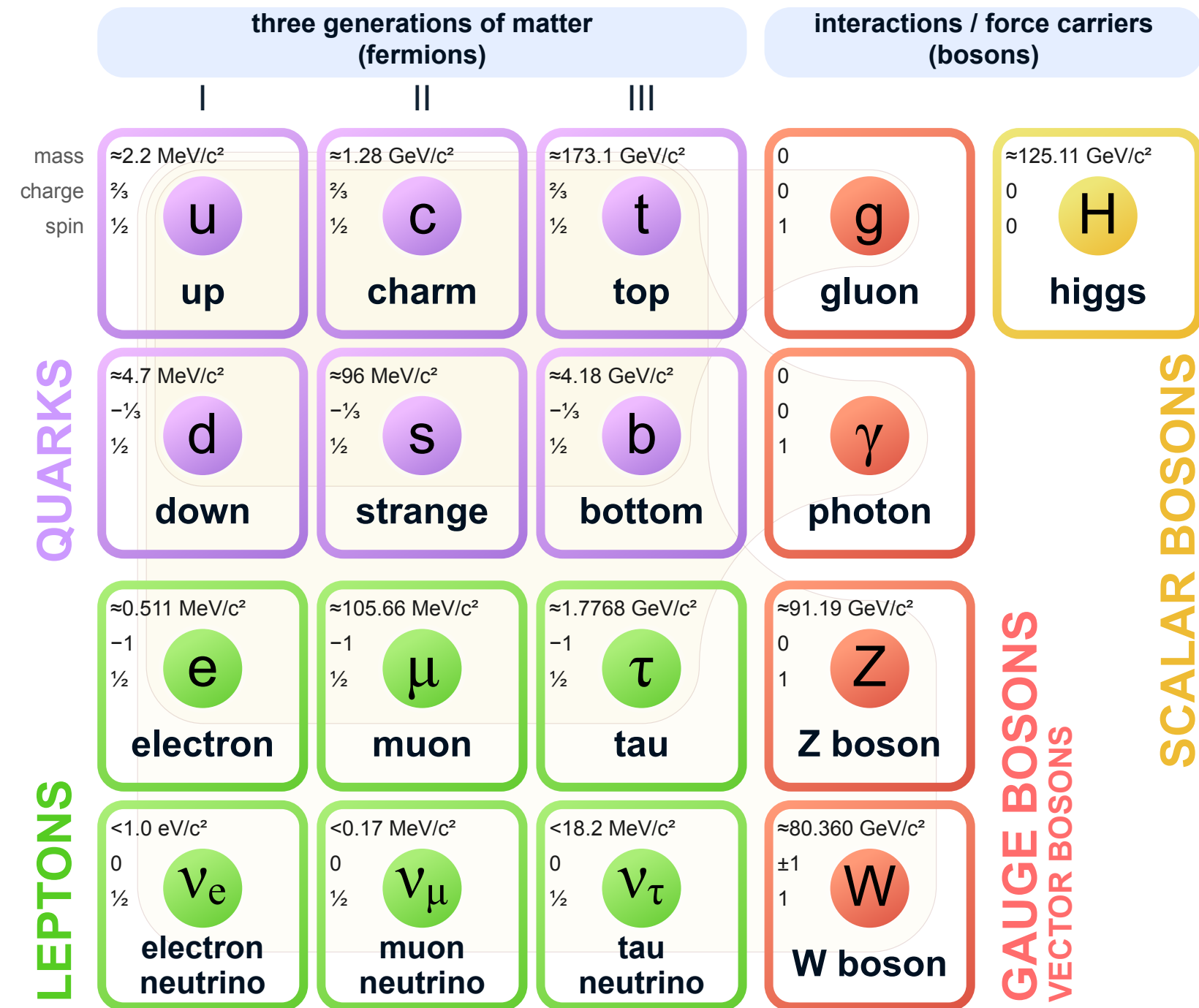All other units (for example volts), are derived from the base units.

Electrons

Electrons are fundamental, they cannot (as far as we know), be divided into smaller parts. Explained further in the standard model of particle physics

An electron cannot occupy the same quantum state as another. This rule that applies to all Fermions (particles with spin of 1/2)

The quantum state of an electron is fully described by it's spin, momentum (p) and position in space (r).

Standard Model of Elementary Particles

three generations of matter (fermions)			interactions / force carriers (bosons)	
I	II	III		

QUARKS

| ≈2.2 MeV/c² ⅔ ½ **u** up | ≈1.28 GeV/c² ⅔ ½ **c** charm | ≈173.1 GeV/c² ⅔ ½ **t** top | 0 0 1 **g** gluon | ≈125.11 GeV/c² 0 0 **H** higgs |
| ≈4.7 MeV/c² −⅓ ½ **d** down | ≈96 MeV/c² −⅓ ½ **s** strange | ≈4.18 GeV/c² −⅓ ½ **b** bottom | 0 0 1 **γ** photon | |

LEPTONS

| ≈0.511 MeV/c² −1 ½ **e** electron | ≈105.66 MeV/c² −1 ½ **μ** muon | ≈1.7768 GeV/c² −1 ½ **τ** tau | ≈91.19 GeV/c² 0 1 **Z** Z boson | |
| <1.0 eV/c² 0 ½ **ν$_e$** electron neutrino | <0.17 MeV/c² 0 ½ **ν$_μ$** muon neutrino | <18.2 MeV/c² 0 ½ **ν$_τ$** tau neutrino | ≈80.360 GeV/c² ±1 1 **W** W boson | |

mass charge spin

SCALAR BOSONS

GAUGE BOSONS VECTOR BOSONS

Probability

The probability of finding an electron in a state as a function of space and time is

$$P = |\psi(r, t)|^2$$

, where ψ is named the probability amplitude, and is a complex function of space and time. In some special cases, it's

$$\psi(r, t) = Ae^{i(kr - \omega t)}$$

Uncertainty principle

We cannot, with ultimate precision, determine both the position and the momentum of a particle, the precision is

$$\sigma_x \sigma_p \geq \frac{\hbar}{2}$$

From the uncertainty (Unschärfe) principle we can actually estimate the size of the atom

States as a function of time and space

The time-evolution of the probability amplitude is

$$i\hbar\frac{d}{dt}\psi(r,t) = H\psi(r,t)$$

, where H is named the Hamiltonian matrix, or the energy matrix or (if I understand correctly) the amplitude matrix of the probability amplitude to change from one state to another.

Allowed energy levels in atoms

Solutions to Schrodinger result in quantized energy levels for an electron bound to an atom.

The probability of a state transition (change in energy) can be determined from the probability amplitude and Schrodinger.

Allowed energy levels in solids

$$H = \begin{bmatrix} A & 0 \\ 0 & -A \end{bmatrix}$$
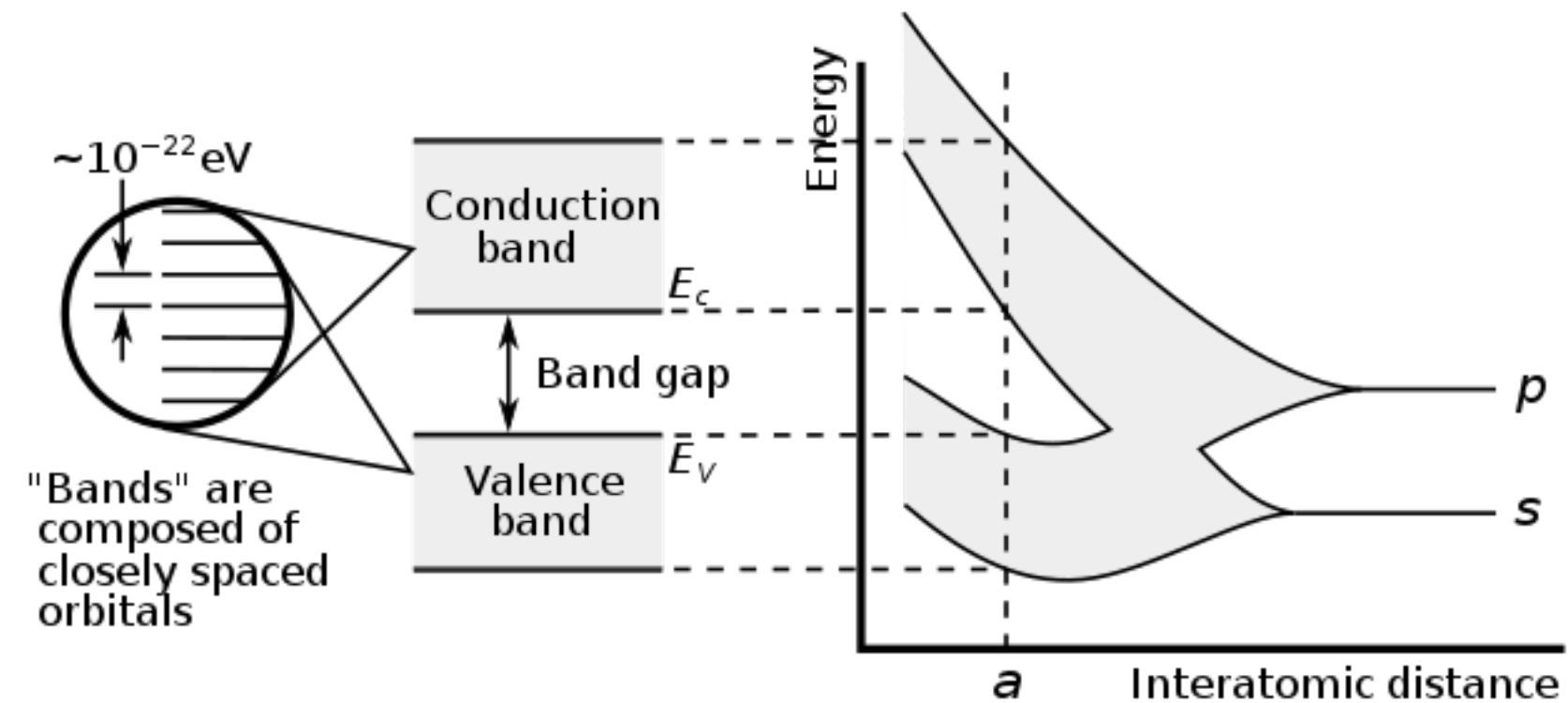
and the new energy levels could be

$$E_1 = E_0 + A$$

and

$$E_2 = E_0 - A$$

The discrete energy levels of the electron transition into bands of allowed energy states.

For a crystal, the allowed energy bands is captured in the band structure

Silicon Unit Cell

A silicon crystal unit cell is a diamond faced cubic with 8 atoms in the corners spaced at 0.543 nm, 6 at the center of the faces, and 4 atoms inside the unit cell at a nearest neighbor distance of 0.235 nm.
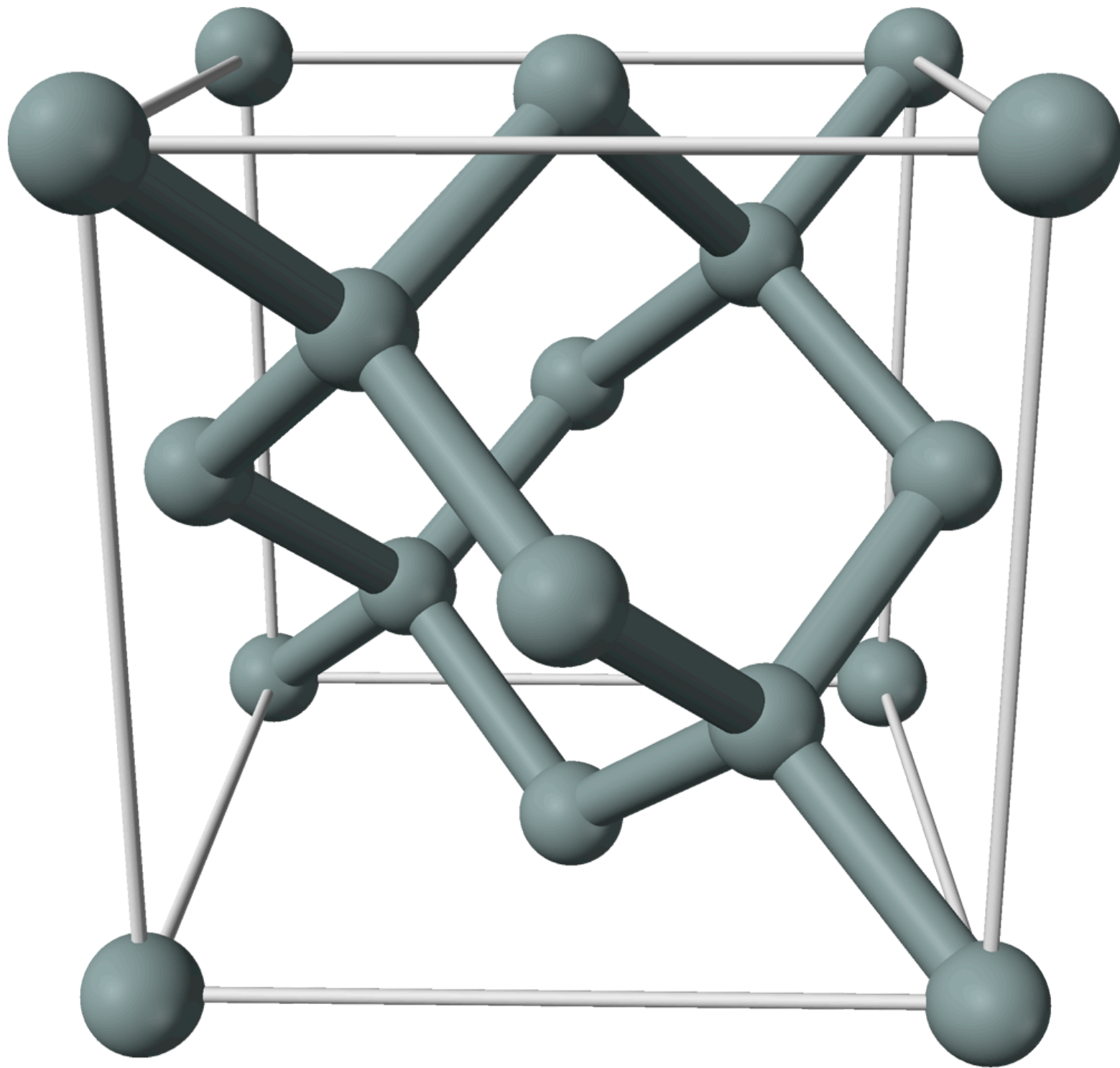
Band structure

The full band structure of a silicon unit cell is complicated, it's a 3 dimensional concept
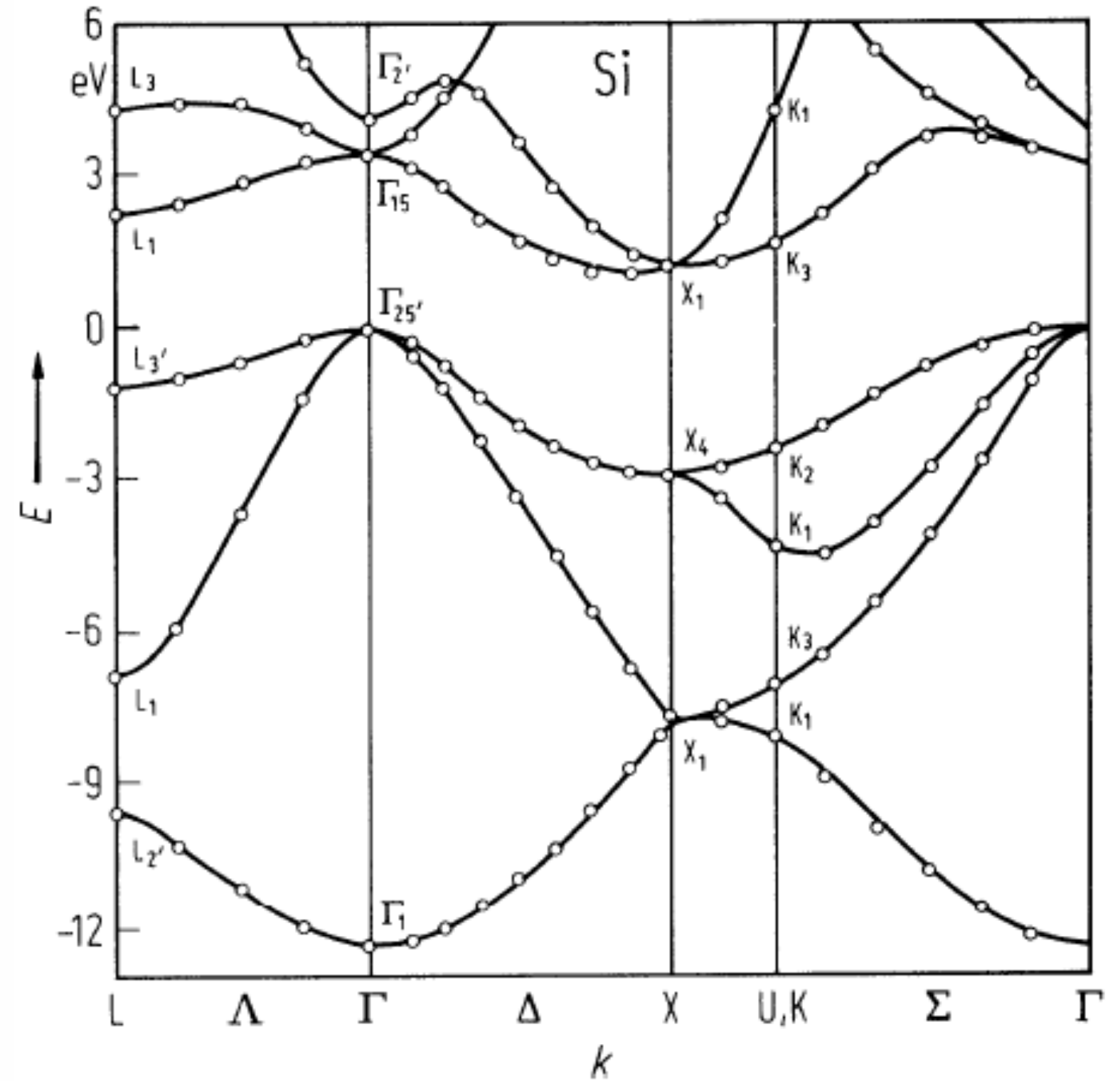
Valence band and Conduction band

For bulk silicon we simplify, and we think of two bands, the conduction band, and valence band
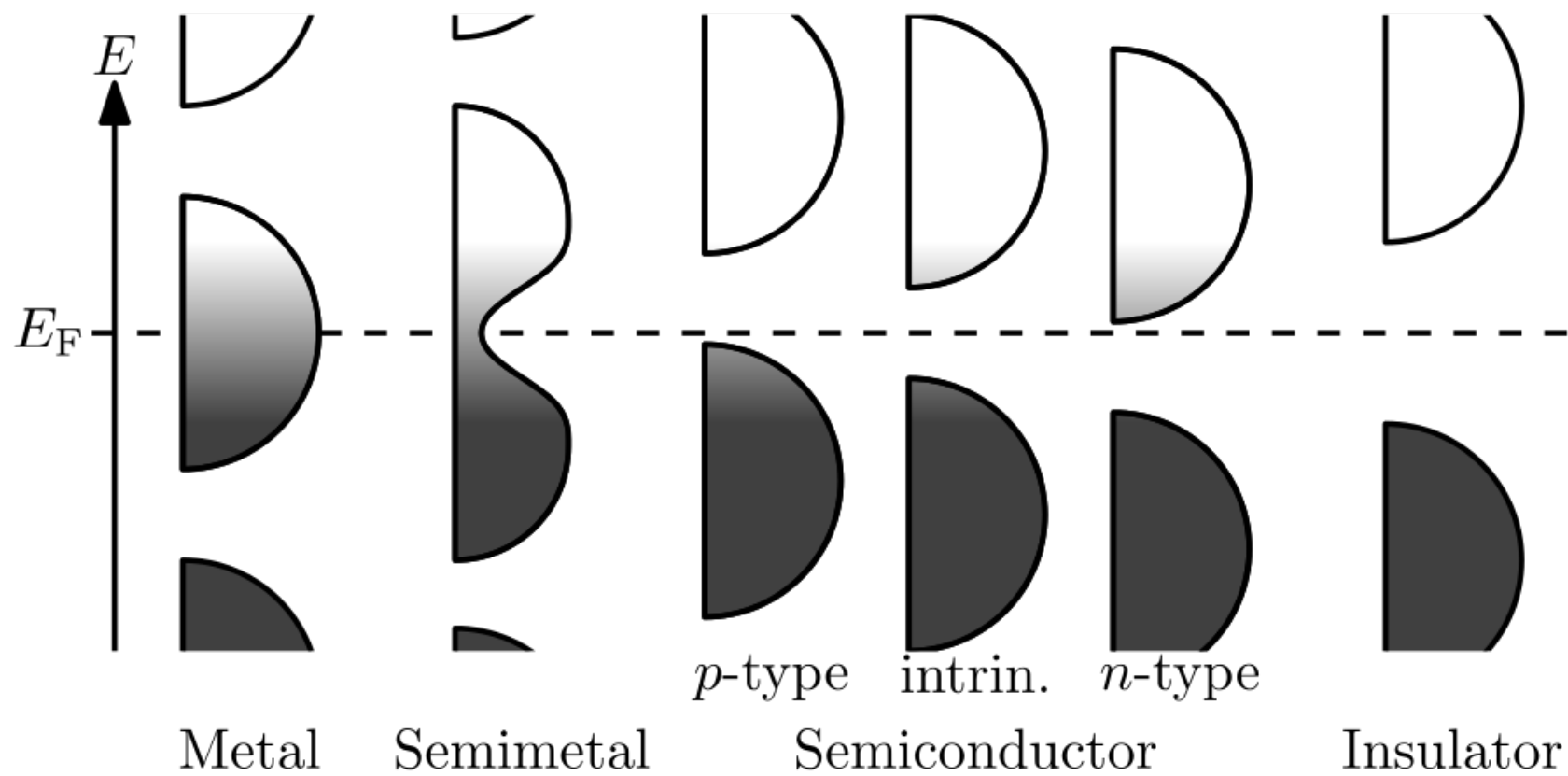
$$E_G = E_C - E_V$$

Fermi level

In band structure theory, used in solid state physics to analyze the energy levels in a solid, the Fermi level can be considered to be a hypothetical energy level of an electron, such that at thermodynamic equilibrium this energy level would have a 50% probability of being occupied at any given time

$$f(E) = \frac{1}{e^{(E-E_F)/kT} + 1}$$

$$f(E) \approx e^{(E_F-E)/kT}$$

Metals

In metals, the band splitting of the energy levels causes the valence band and conduction band to overlap.

Insulators

In insulating materials the difference between the conduction band and the valence band is large. As a result, it takes a large energy to excite electrons to a state where they can freely move.

That's why glass is transparent to optical frequencies. Visible light does not have sufficient energy to excite electrons from a bound state.

That's also why glass is opaque to ultra-violet, which has enough energy to excite electrons out of a bound state.

Based on these two pieces of information you could estimate the bandgap of glass.

Semiconductors

In a silicon the bandgap is lower than an insulator, approximately

$$E_G = 1.12\ eV$$

At room temperature, that allows a small number of electrons to be excited into the conduction band, leaving behind a "hole" in the valence band.

Band diagrams

A band diagram or energy level
diagrams shows the conduction band
energy and valence band energy as a
function
of distance in the material.

The horizontal axis is the distance, the
vertical axis is the energy.

The figure shows a PN-junction
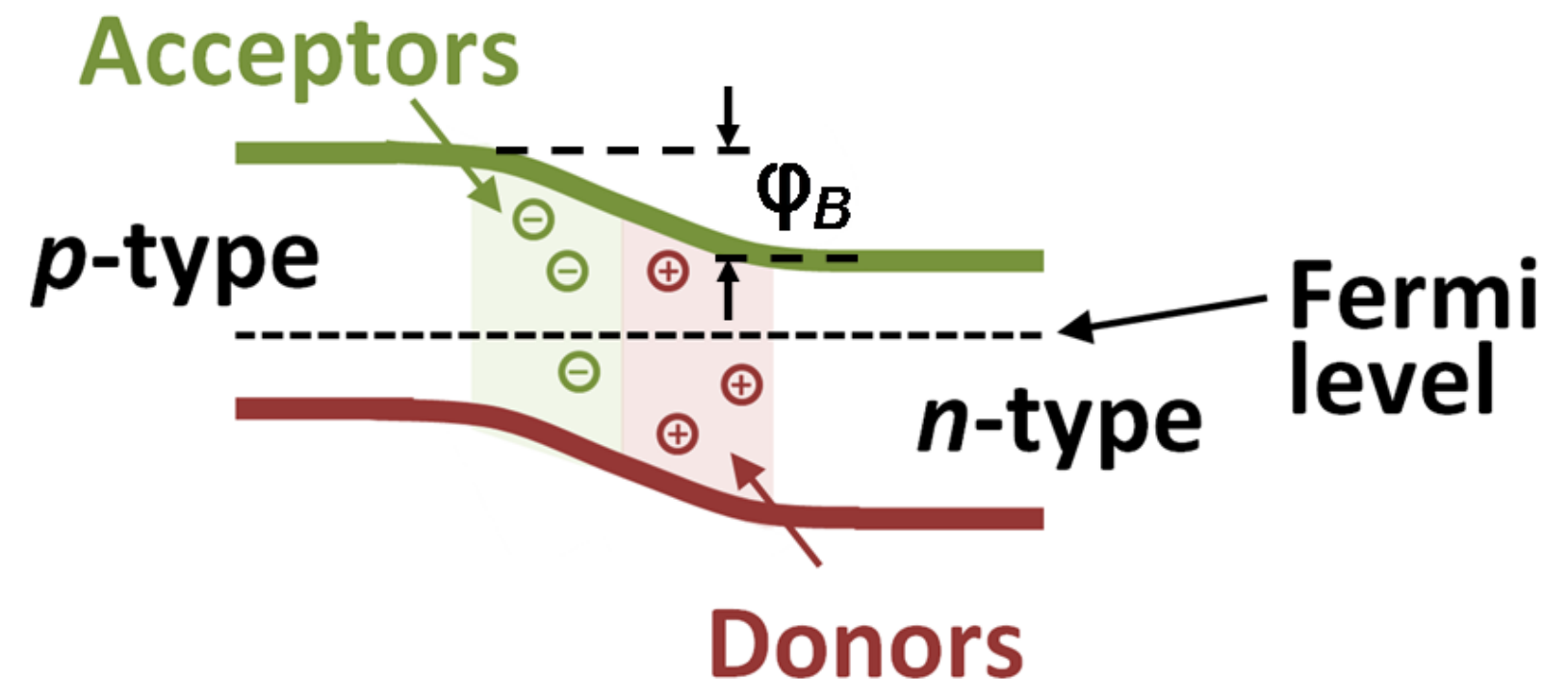
Density of electrons/holes

$$n_e = \int_{E_C}^{\infty} N(E) f(E) dE$$

$$n_e = e^{E_F/kT} \int_{E_C}^{\infty} N(E) e^{-E/kT} dE$$

Fields

There are equations that relate electric field, magnetic field, charge density and current density to each-other.

$$\oint_{\partial\Omega} \mathbf{E} \cdot d\mathbf{S} = \frac{1}{\epsilon_0} \iiint_V \rho \cdot dV$$

,relates net electric flux to net enclosed electric charge

$$\oint_{\partial\Omega} \mathbf{B} \cdot d\mathbf{S} = 0$$

,relates net magnetic flux to net enclosed magnetic charge

$$\oint_{\partial\Sigma} \mathbf{E} \cdot d\ell = -\frac{d}{dt} \iint_{\Sigma} \mathbf{B} \cdot d\mathbf{S}$$

,relates induced electric field to changing magnetic flux

$$\oint_{\partial \Sigma} \mathbf{B} \cdot d\ell = \mu_0 \left(\iint_{\Sigma} \mathbf{J} \cdot d\mathbf{S} + \epsilon_0 \frac{d}{dt} \iint_{\Sigma} \mathbf{E} \cdot d\mathbf{S} \right)$$

,relates induced magnetic field to changing electric flux and to current

These are the Maxwell Equations, and are non-linear time dependent differential equations.

Under the best of circumstances they are fantastically hard to solve! But it's how the real world works.

The permittivity of free space is defined as

$$\epsilon_0 = \frac{1}{\mu_0 c^2}$$

, where c is the speed of light, and μ_0 is the vacuum permeability, which, in SI units, is now

$$\mu_0 = \frac{2\alpha}{q^2} \frac{h}{c}$$

, where α is the fine structure constant.

Voltage

The electric field has units voltage per meter, so the electric field is the derivative of the voltage as a function of space.

$$E = \frac{dV}{dx}$$

Current

Current has unit A and charge C has unit As, so the current is the number of charges passing through a volume per second.

The current density J has units A/m^2 and is often used, since we can multiply by the surface area of a conductor, if the current density is uniform.

$$I = A \times J$$

Drift current

Charges in an electric field will give rise to a drift current.

We know from Newtons laws that force equals mass times acceleration

$$\vec{F} = m\vec{a}$$

If we assume a zero, or constant magnetic field, the force on a particle is

$$\vec{F} = q\vec{E}$$

The current density is then

$$\vec{J} = q\vec{E} \times n \times \mu$$

Assuming

$$E = V/m$$

, we could write

$$J = \frac{C}{m^3} \frac{V}{m} \frac{m^2}{Vs} = \frac{C}{s} m^{-2}$$

So multiplying by an area

$$A = Bm^2$$

$$I = qn\mu BV$$

and we can see that the conductance

$$G = qn\mu B$$

, and since

$$G = 1/R$$

, where R is the resistance, we have

$$I = GV \Rightarrow V = RI$$

Or Ohms law

Diffusion current

A difference in charge density will give rise to a diffusion current, and the current density is

$$J = -qD_n \frac{d\rho}{dx}$$

Why are there two currents?

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial^2 x}\psi(x,t) + V(x)\psi(x,t) = i\hbar\frac{\partial}{\partial t}\psi(x,t)$$

Currents in a semiconductor

Both electrons, and holes will contribute to current.

Electrons move in the conduction band, and holes move in the valence band.

$$I = I_{n_{drift}} + I_{n_{diffusion}} + I_{p_{drift}} + I_{p_{diffusion}}$$

Resistors

We can make resistors with metal and silicon (a semiconductor)

In metal the dominant carrier depends on the metal, but it's usually electrons. As such, one can often ignore the hole current.

In a semiconductor the dominant carrier depends on the Fermi level in relation to the conduction band and valence band. If the Fermi level is close to the valence band the dominant carrier will be holes. If the Fermi level is close to the conduction band, the dominant carrier will be electrons.

That's why we often talk about "majority carriers" and "minority carriers", both are important in semiconductors.

Capacitors

A capacitor resists a change in voltage.

$$I = C\frac{dV}{dt}$$

and store energy in an electric field between two conductors with an insulator between.

Inductors

An inductor resist a change in current.

$$V = L\frac{dI}{dt}$$

and store energy in the magnetic fields in a loop of a conductor.