

# TFE4188 - Introduction to Lecture 5

# Switched-Capacitor Circuits

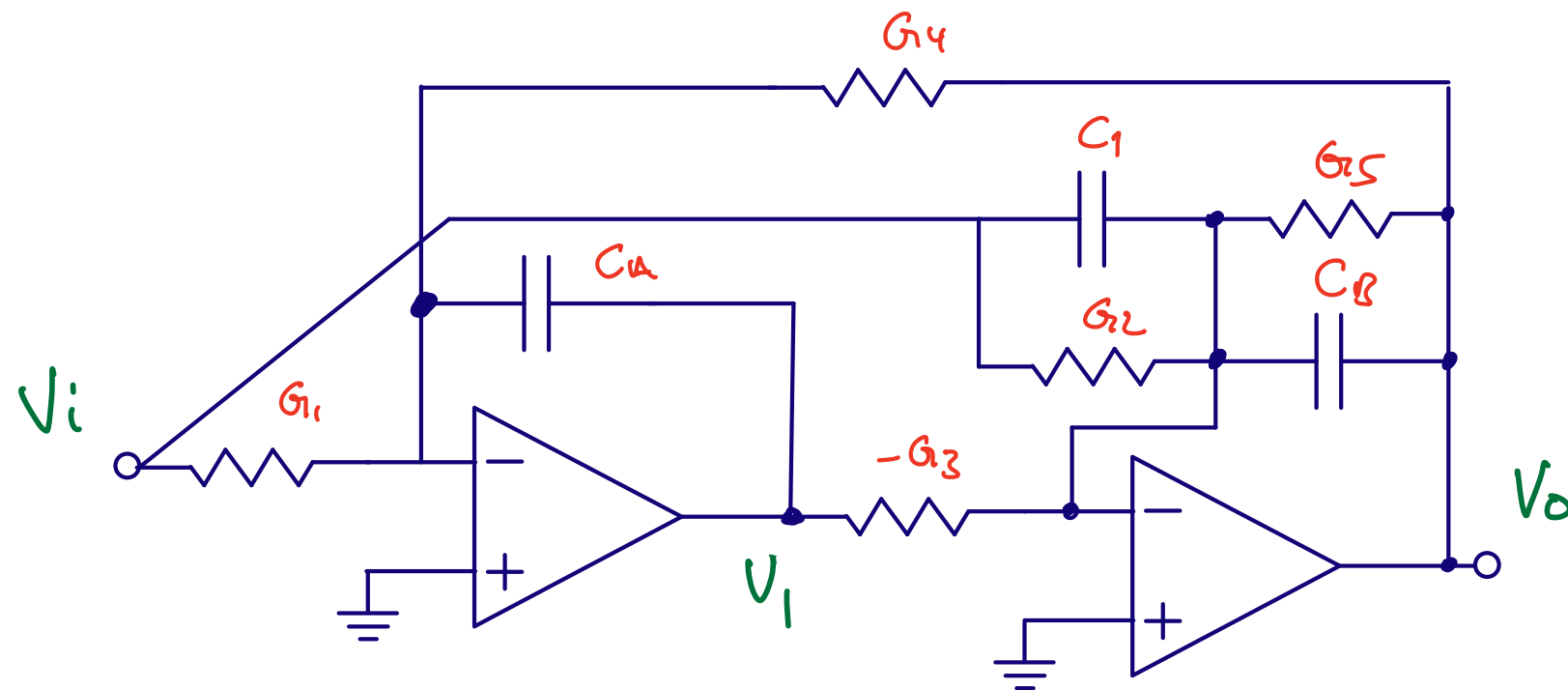
# Goal

Understand **why** we would use switched capacitor circuits

Introduction to **discrete-time**, and **switched capacitor principles**  
and the **circuits** we need

**W w h y**

# Active-RC



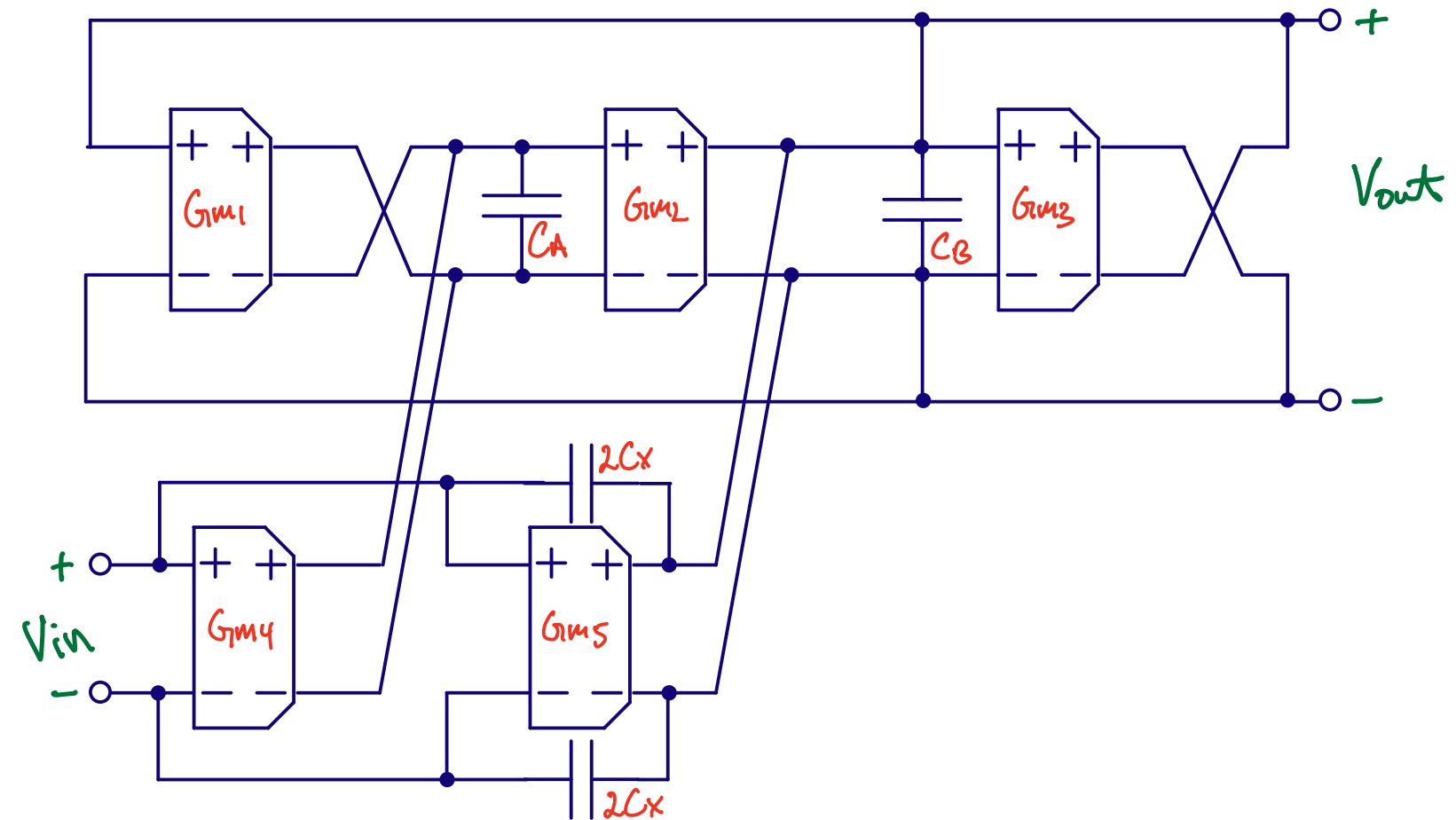
$$H(s) = \frac{\left[ \frac{C_1}{C_B} s^2 + \frac{G_2}{C_B} s + \left( \frac{G_1 G_3}{C_A C_B} \right) \right]}{\left[ s^2 + \frac{G_5}{C_B} s + \frac{G_3 G_4}{C_A C_B} \right]}$$

$$\omega_{p|z} \propto \frac{G}{C} = \frac{1}{RC}$$

# Gm-C

$$H(s) = \frac{\left[ s^2 \frac{C_X}{C_X + C_B} + s \frac{G_{m5}}{C_X + C_B} + \frac{G_{m2} G_{m4}}{C_A (C_X + C_B)} \right]}{\left[ s^2 + s \frac{G_{m2}}{C_X + C_B} + \frac{G_{m1} G_{m2}}{C_A (C_X + C_B)} \right]}$$

$$\omega_{p|z} \propto \frac{G_m}{C}$$



# Switched capacitor circuits

A pipelined 5-Msample/s 9-bit analog-to-digital converter

$$\omega_{p|z} \propto \frac{C_1}{C_2}$$

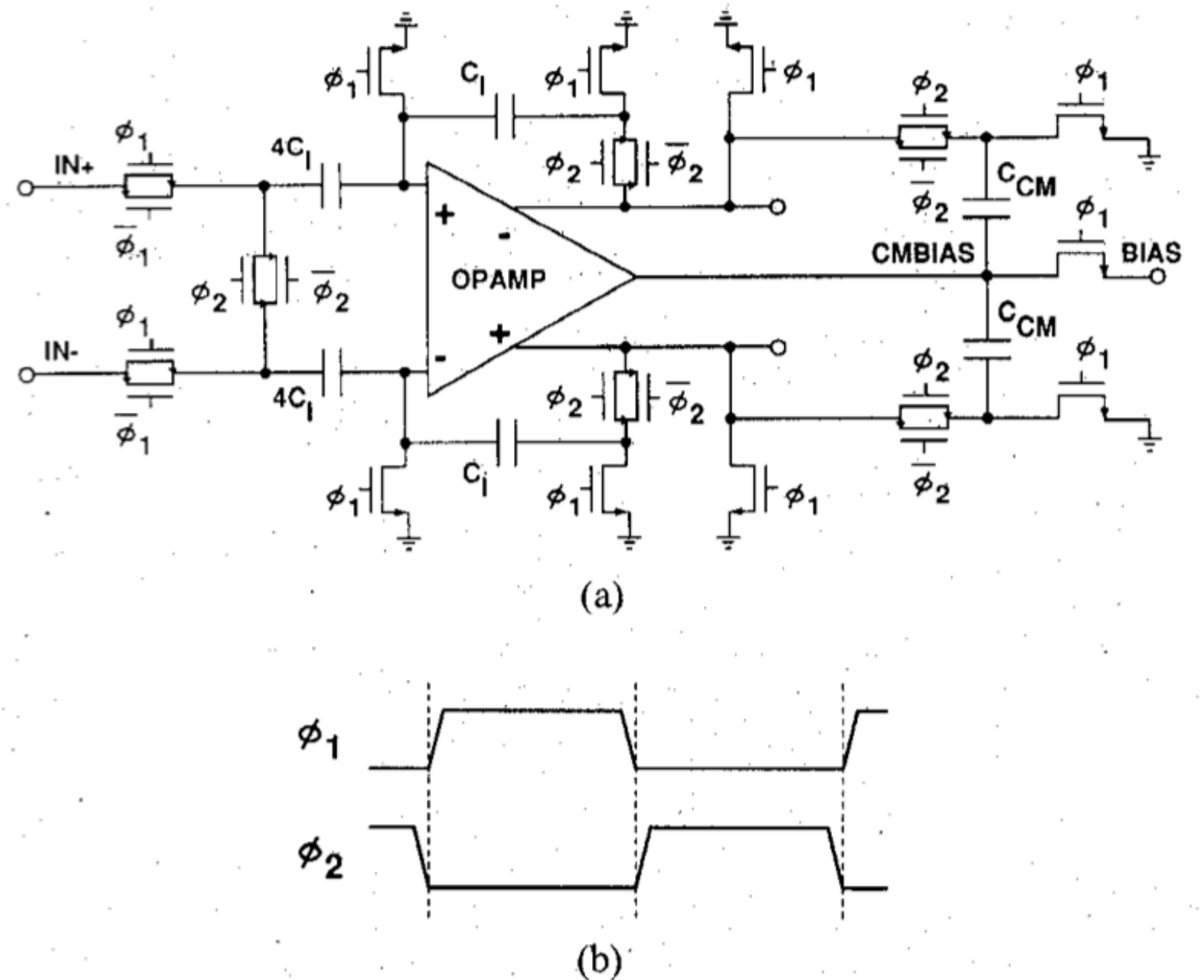


Fig. 6. (a) Schematic of S/H amplifier. (b) Timing diagram of a two-phase nonoverlapping clock.

# Discrete-Time Signals

Define  $x_c$  as a continuous time, continuous value signal

$$\text{Define } \ell(t) = \begin{cases} 1 & \text{if } t \geq 0 \\ 0 & \text{if } t < 0 \end{cases}$$

Define

$$x_{sn}(t) = \frac{x_c(nT)}{\tau} [\ell(t - nT) - \ell(t - nT - \tau)]$$

$$\text{Define } x_s(t) = \sum_{n=-\infty}^{\infty} x_{sn}(t)$$

Think of a sampled version of an analog signal as an infinite sum of pulse trains where the area under the pulse train is equal to the analog signal.

**Why do this?**



$$\text{If } x_s(t) = \sum_{n=-\infty}^{\infty} x_{sn}(t)$$

$$\text{Then } X_{sn}(s) = \frac{1}{\tau} \frac{1 - e^{-s\tau}}{s} x_c(nT) e^{-snT}$$

$$\text{And } X_s(s) = \frac{1}{\tau} \frac{1 - e^{-s\tau}}{s} \sum_{n=-\infty}^{\infty} x_c(nT) e^{-snT}$$

$$\text{Thus } \lim_{\tau \rightarrow 0} X_s(s) = \sum_{n=-\infty}^{\infty} x_c(nT) e^{-snT}$$

$$\text{Or } X_s(j\omega) = \frac{1}{T} \sum_{k=-\infty}^{\infty} X_c \left( j\omega - \frac{jk2\pi}{T} \right)$$

**The spectrum of a sampled signal is an infinite sum of frequency shifted spectra**

or equivalently

**When you sample a signal, then there will be copies of the input spectrum at every  $nf_s$**

However, if you do an FFT of a sampled signal, then all those infinite spectra will fold down between  $0 \rightarrow f_{s1}/2$  or  $-f_{s1}/2 \rightarrow f_{s1}/2$  for a complex FFT

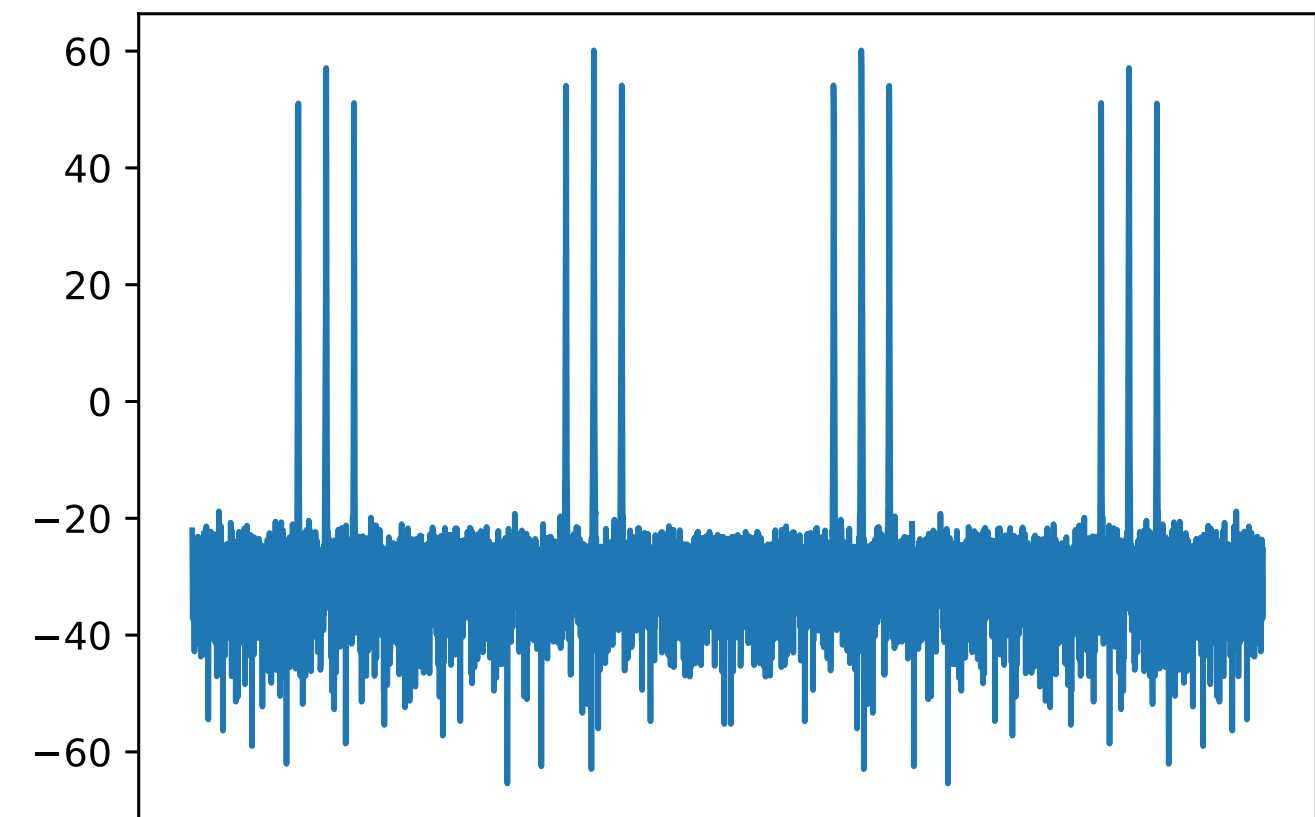
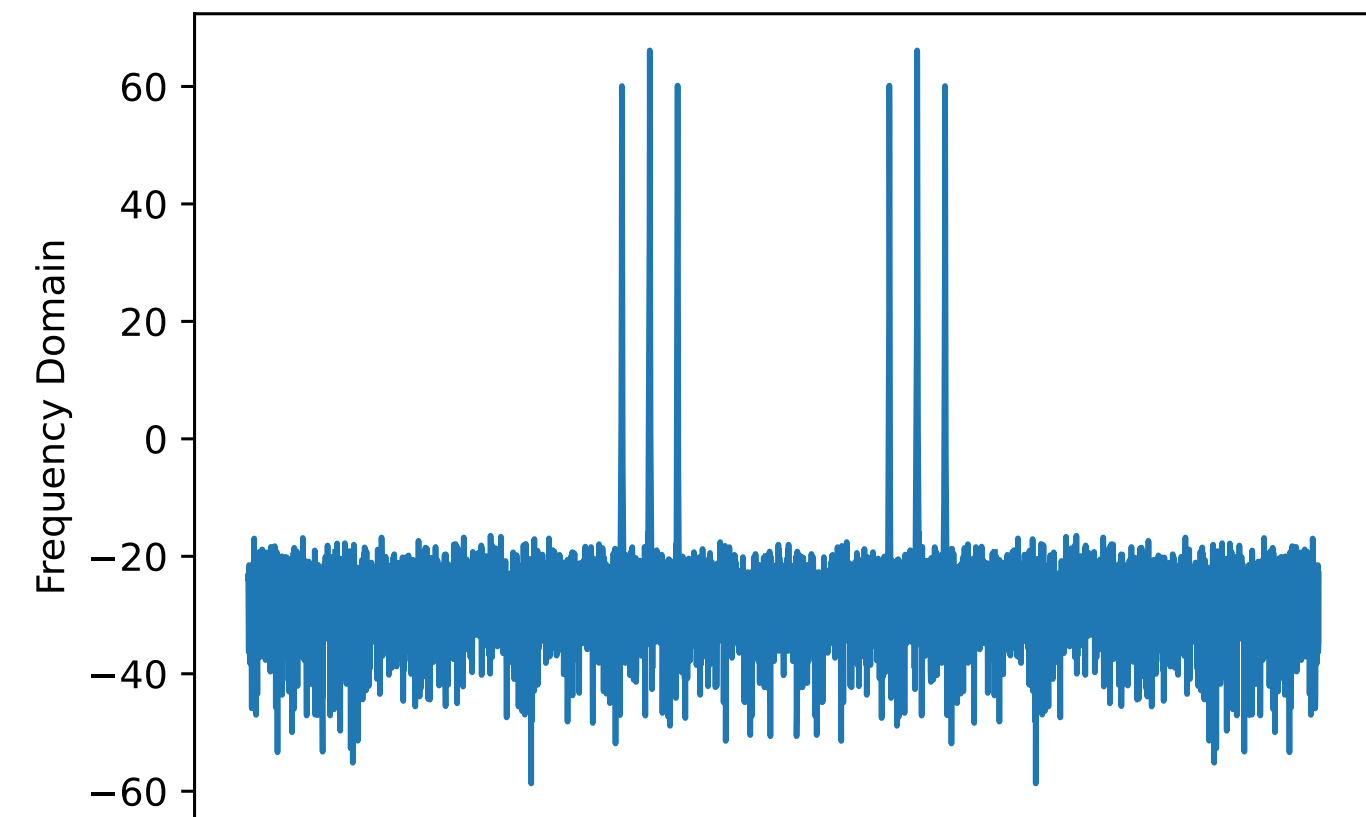
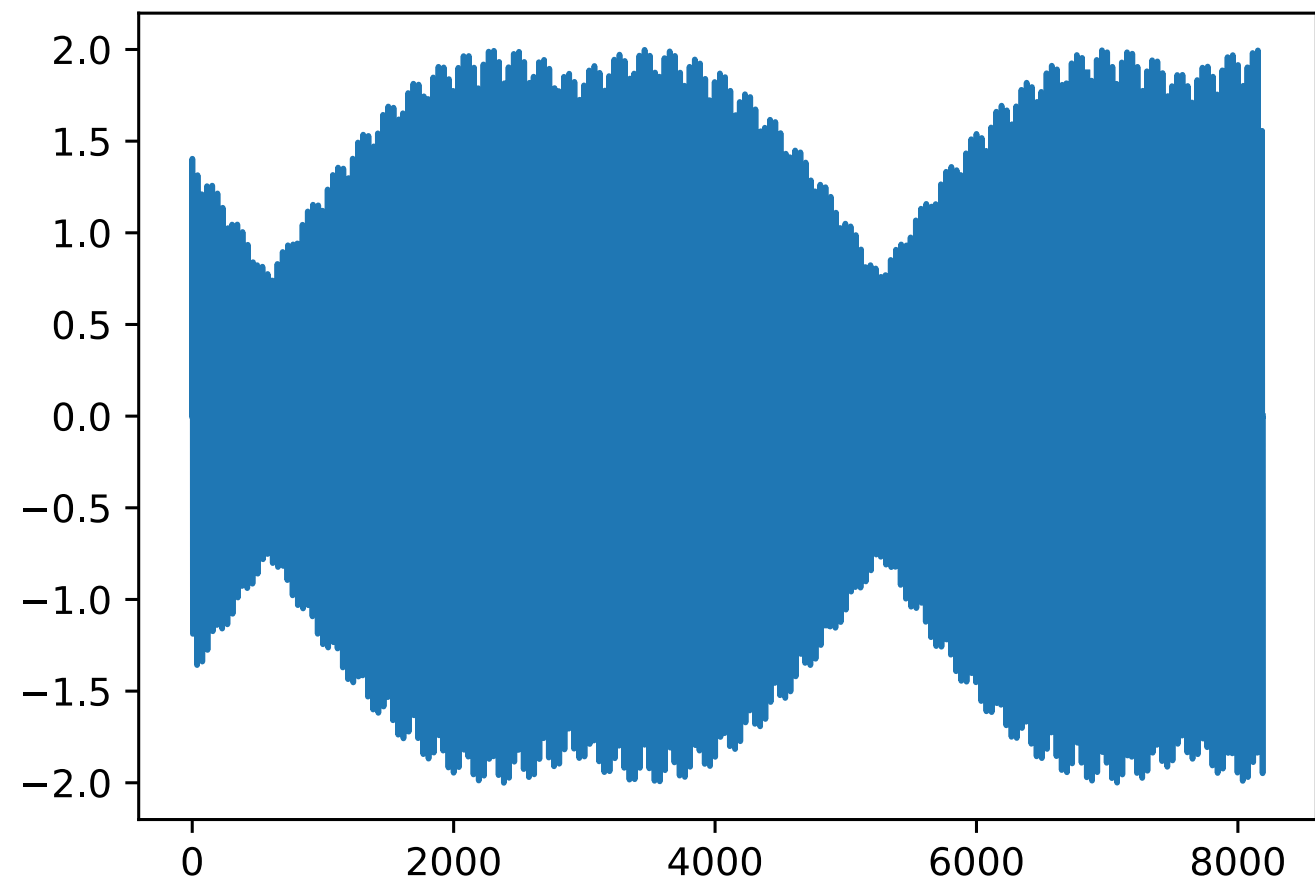
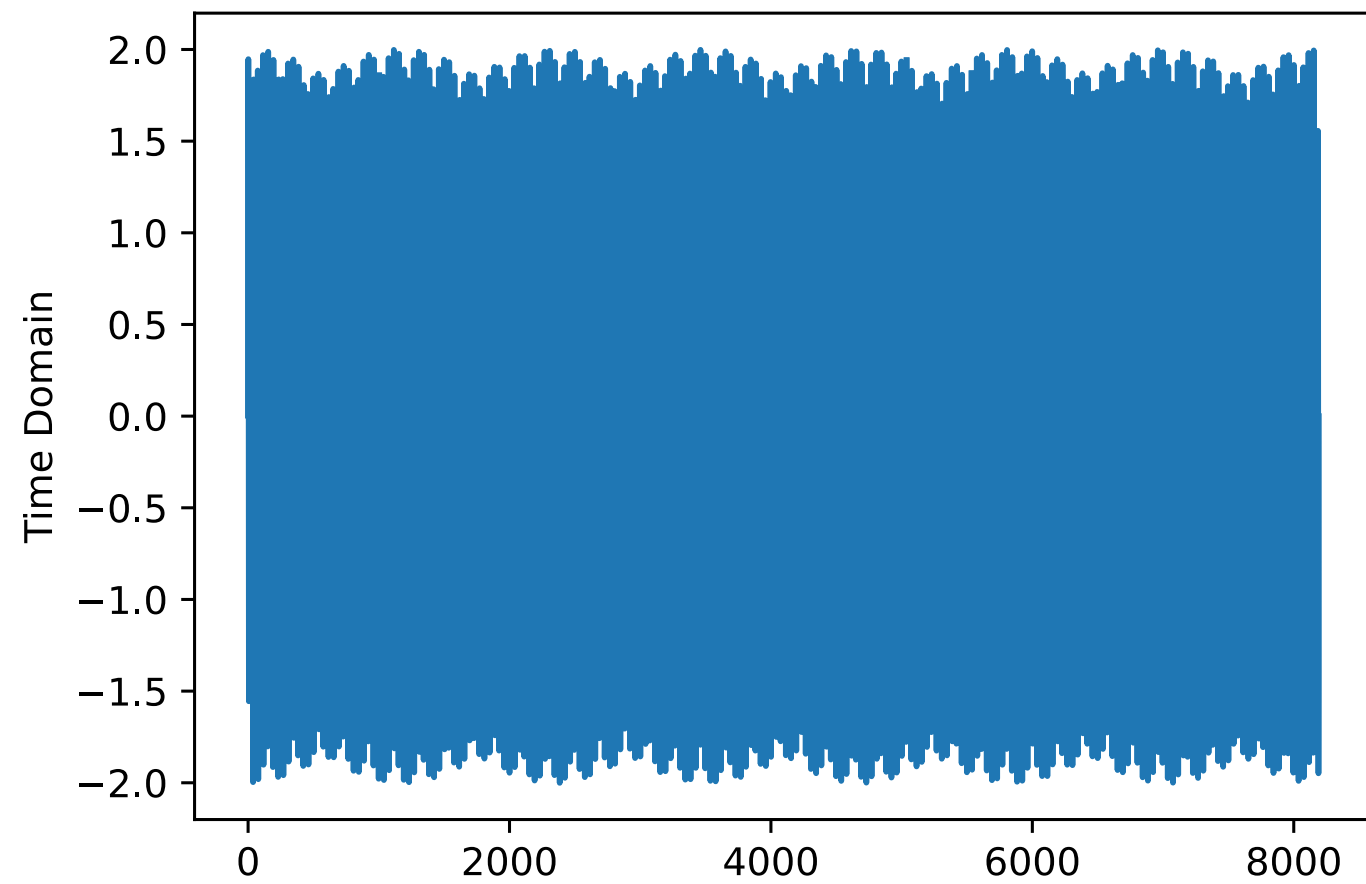
# dt.py

```
#- Create a time vector
N = 2**13
t = np.linspace(0,N,N)

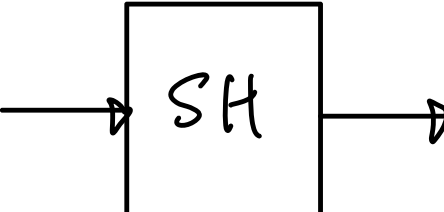
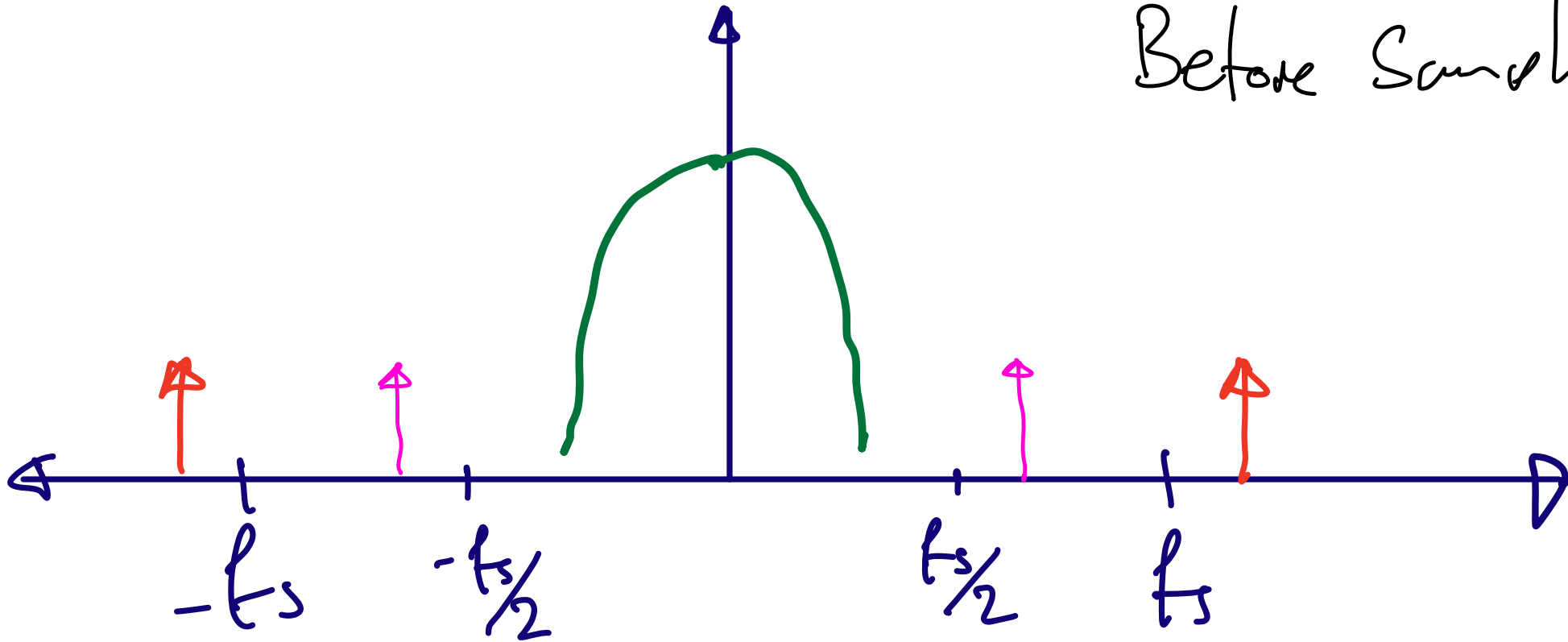
#- Create the "continuous time" signal with multiple sinusoidal signals and some noise
fbin = 10
fm1 = 1/N*213
f1 = 1/8 - 1/N
fd = fm1
x_s = np.sin(2*np.pi*f1*t) + 0.5*np.sin(2*np.pi*(f1-fd)*t) + 0.5*np.sin(2*np.pi*(f1+fd)*t) + 1/1024*np.random.randn(N)

#- Create the sampling vector, and the sampled signal
t_s_unit = [1,1,0,0]
t_s = np.tile(t_s_unit,int(N/len(t_s_unit)))
x_sn = x_s*t_s

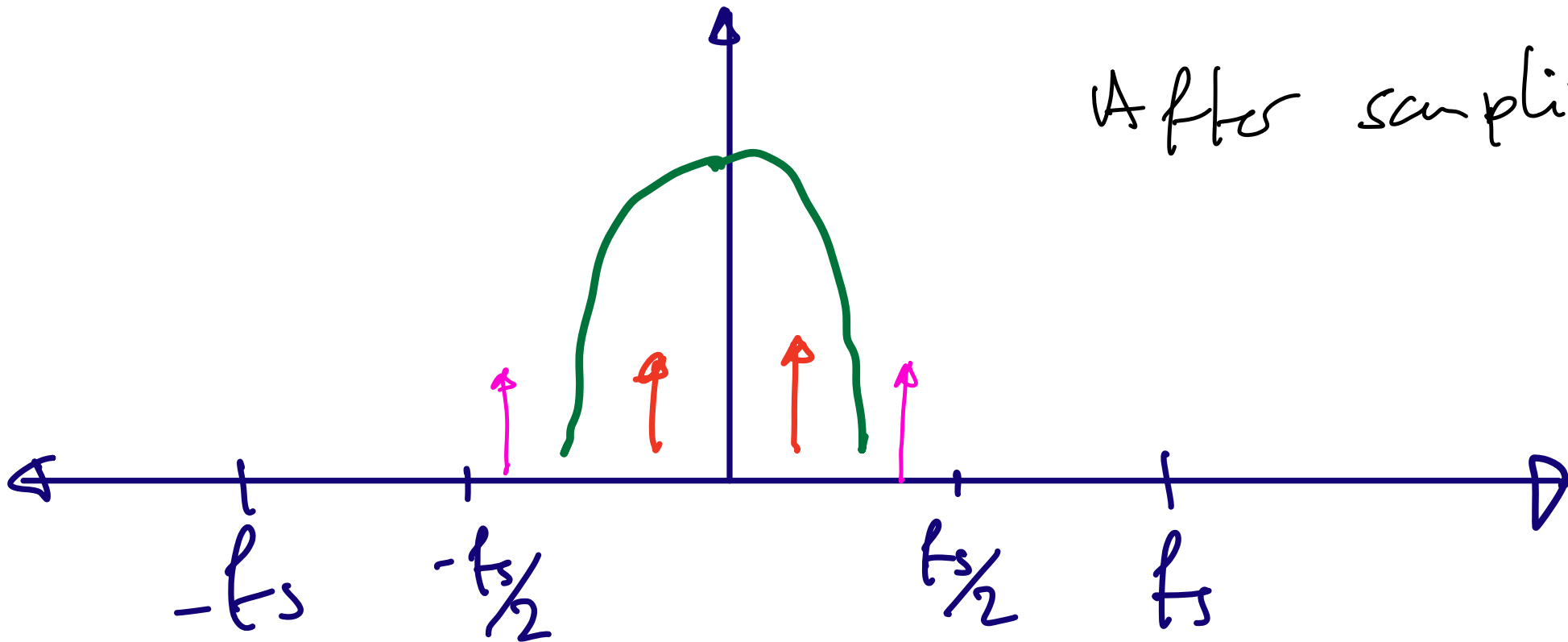
#- Convert to frequency domain with a hanning window to avoid FFT bin
#- energy spread
w = np.hanning(N+1)
X_s = np.fft.fftshift(np.fft.fft(np.multiply(w[0:N],x_s)))
X_sn = np.fft.fftshift(np.fft.fft(np.multiply(w[0:N],x_sn)))
```



Before Sampling

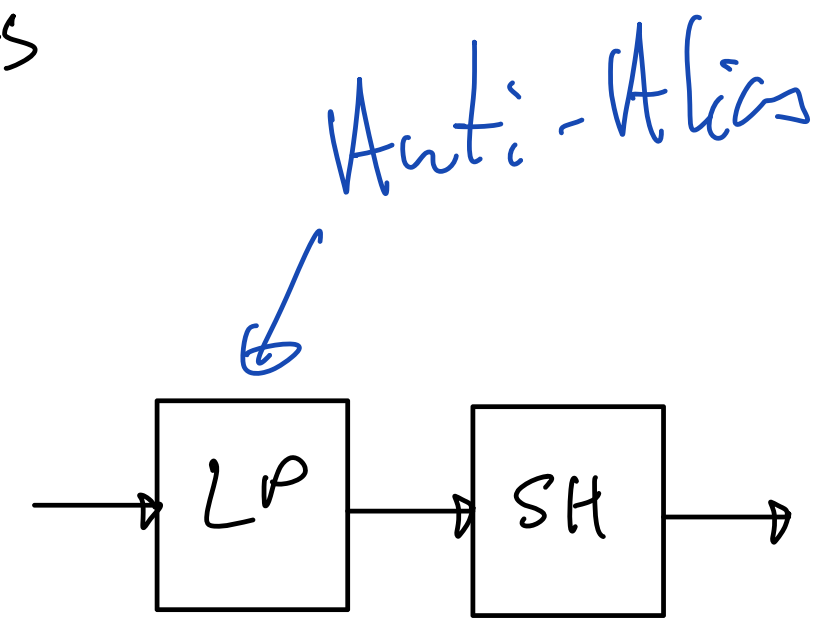
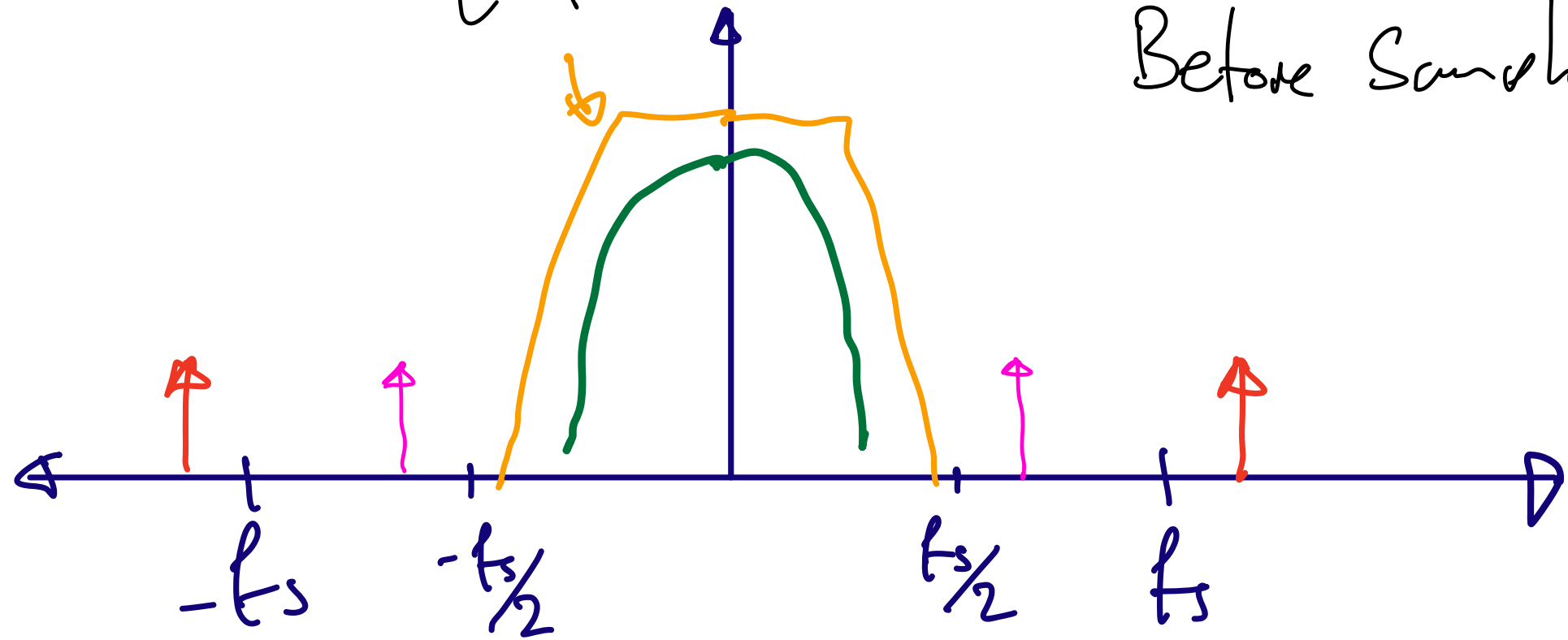


After sampling

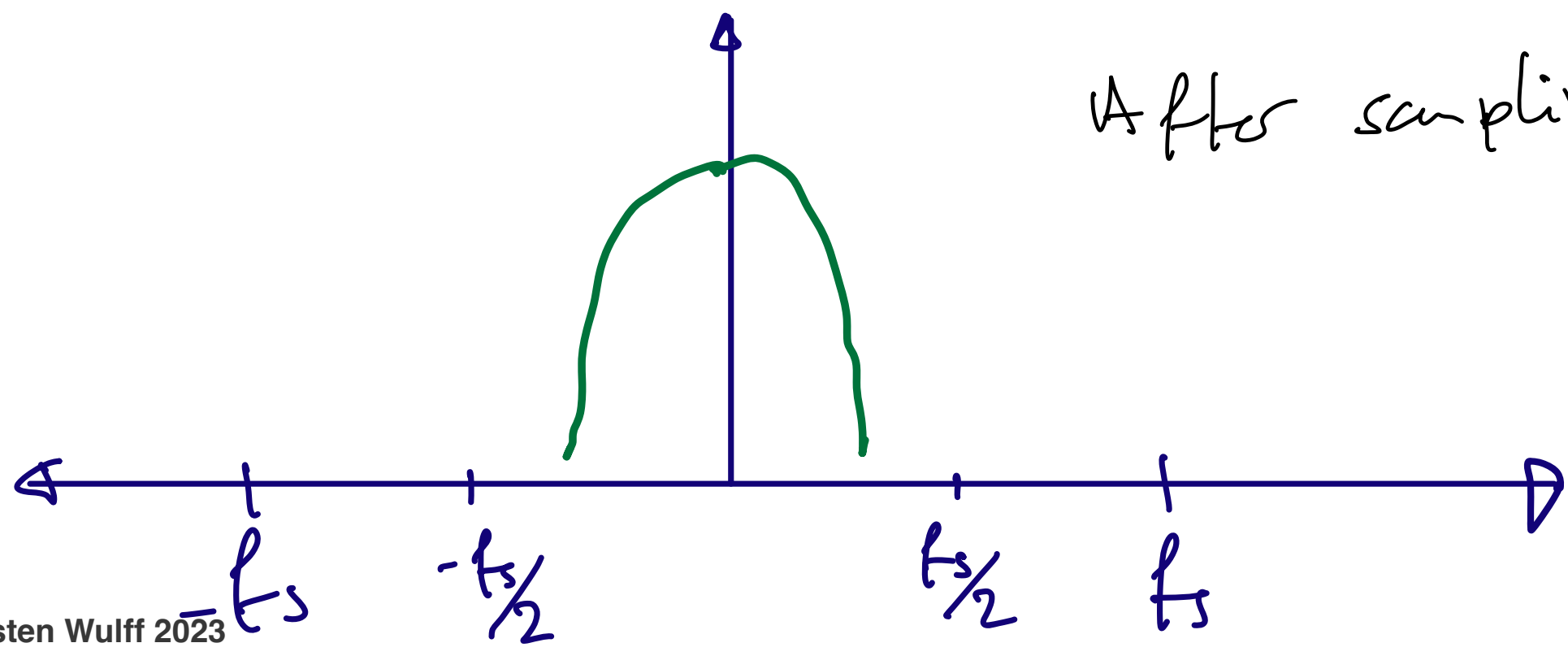


LP filter

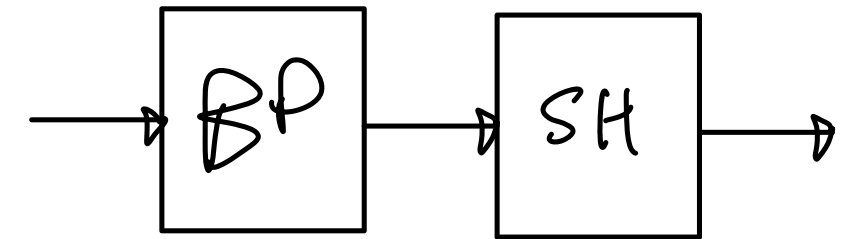
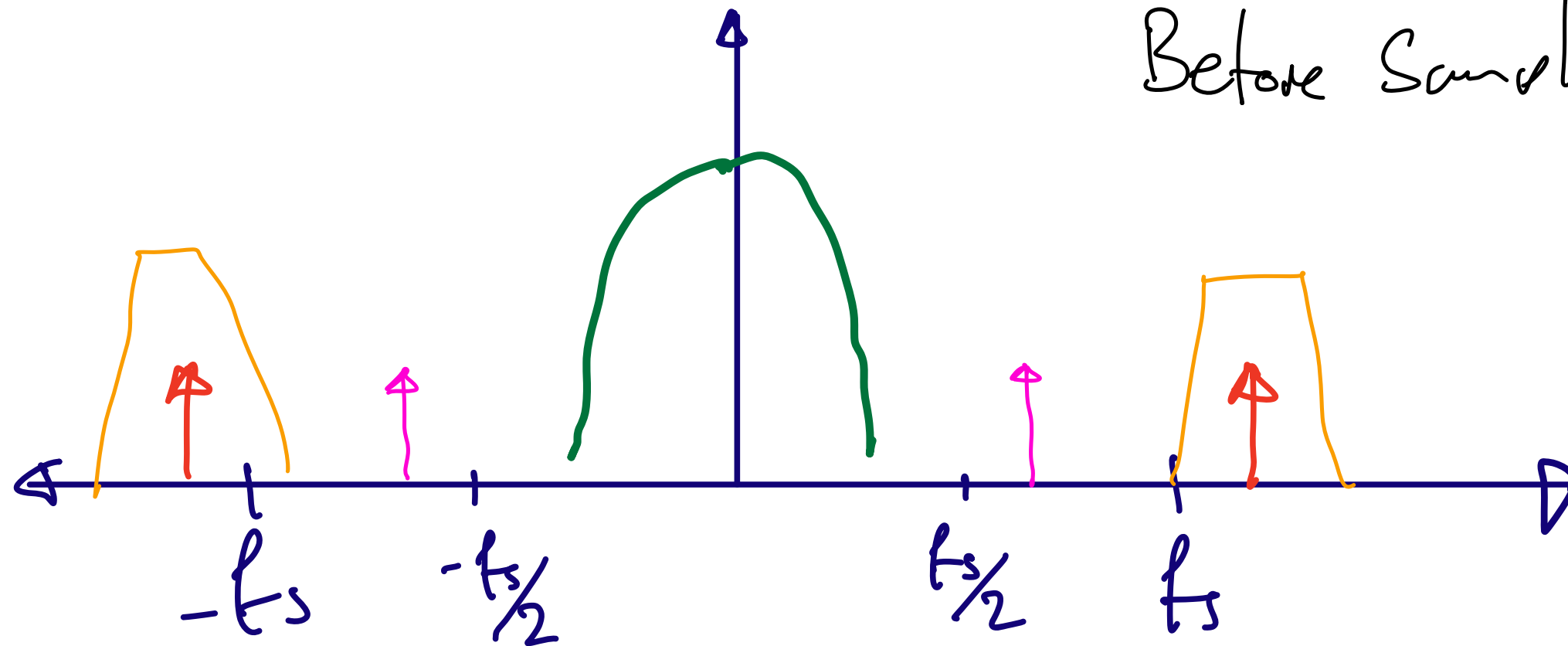
Before Sampling



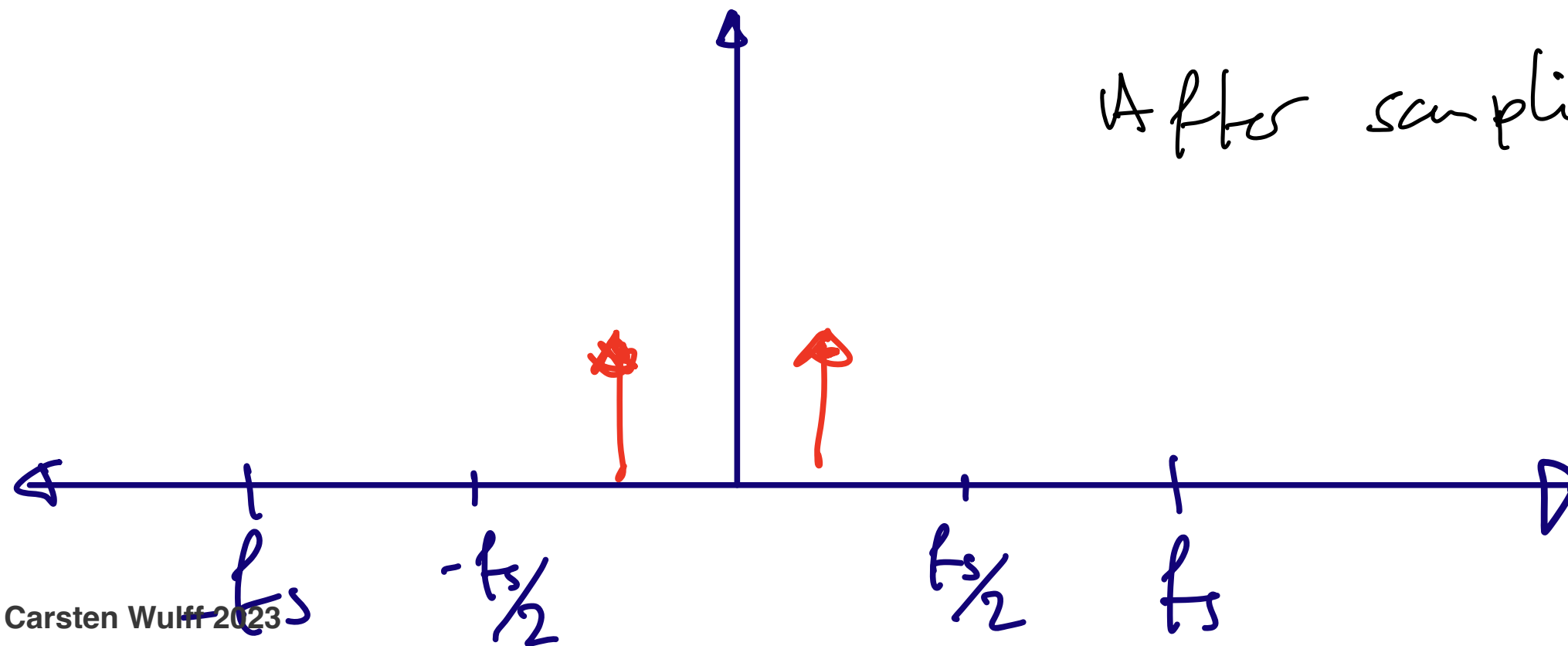
After sampling



Before Sampling



After sampling



$$X_s(s) = \sum_{n=-\infty}^{\infty} x_c(nT) e^{-snT}$$

$$X_s(z) = \sum_{n=-\infty}^{\infty} x_c[n] z^{-n}$$

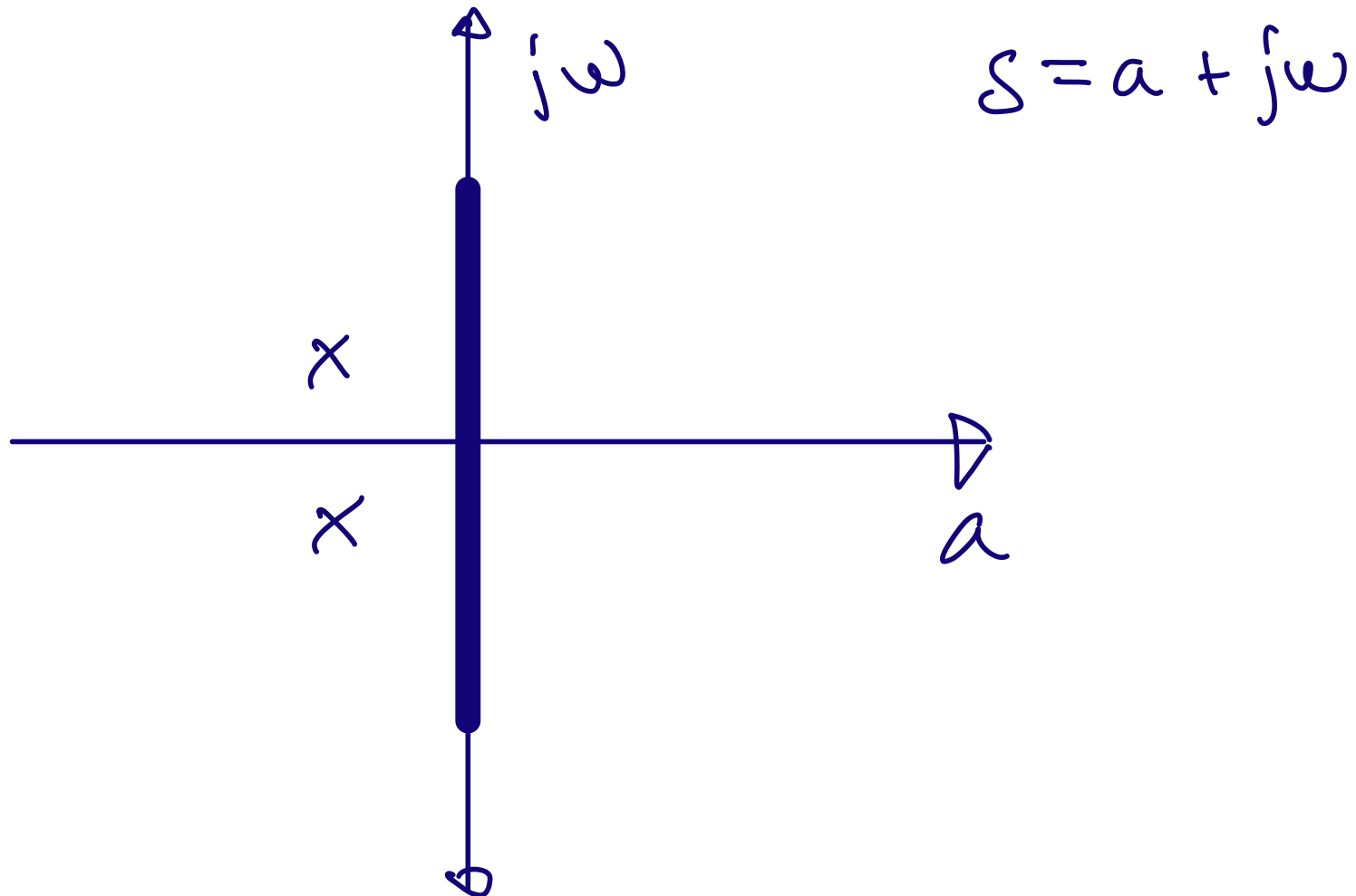
For discrete time signal processing we use Z-transform

If you're unfamiliar with the Z-transform, read the book or search <https://en.wikipedia.org/wiki/Z-transform>

# Pole-Zero plots

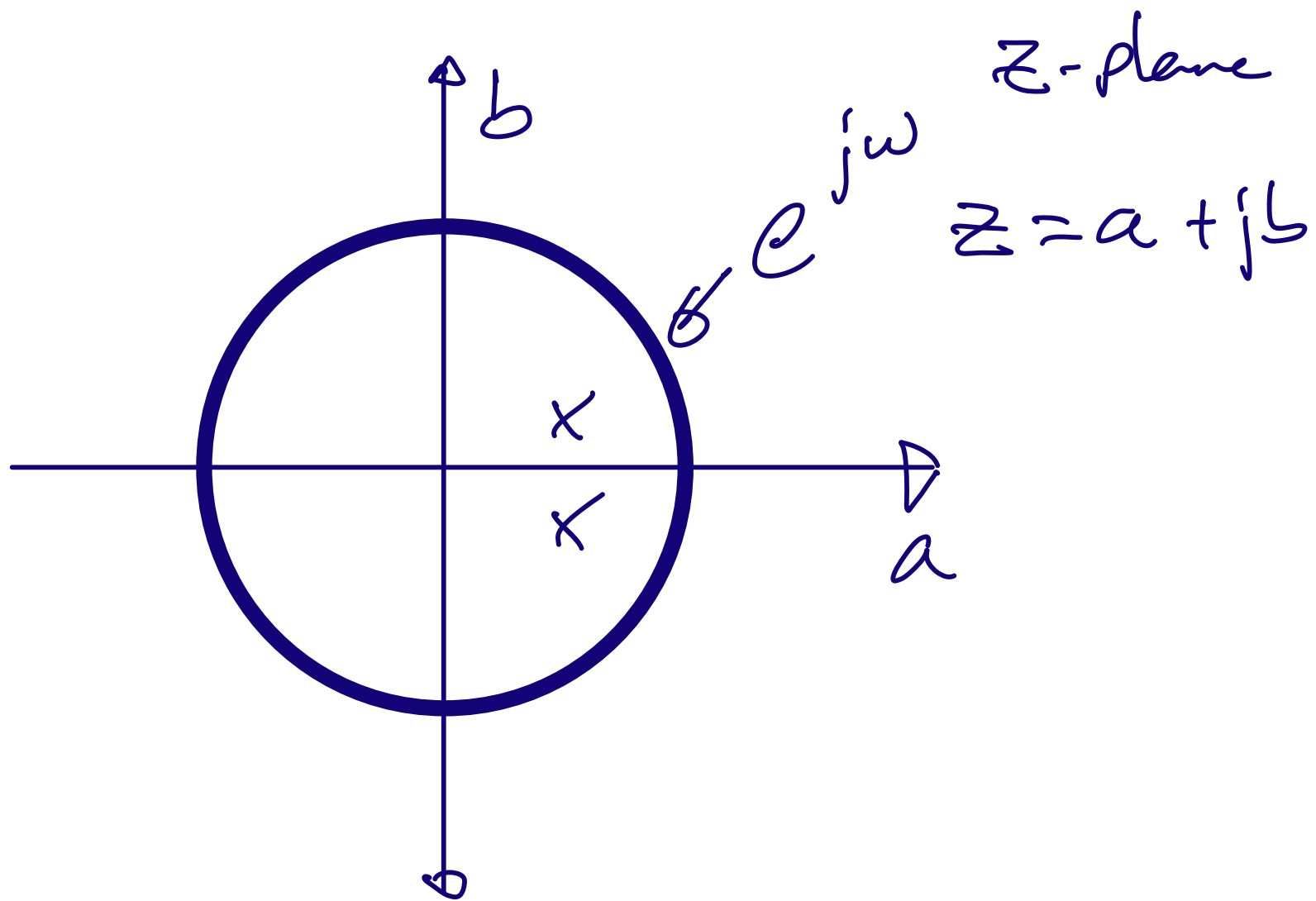
If you're not comfortable with pole/zero plots, have a look at

What does the Laplace Transform really tell us





# Z-domain



Spectra repeat every  $2\pi$  (every  $f_s$ )

Bi-linear transform

$$s = \frac{z - 1}{z + 1}$$

Warning: First-order approximation [https://en.wikipedia.org/wiki/Bilinear\\_transform](https://en.wikipedia.org/wiki/Bilinear_transform)

# First order filter

$$y[n + 1] = bx[n] + ay[n] \Rightarrow yz = bx + ay$$

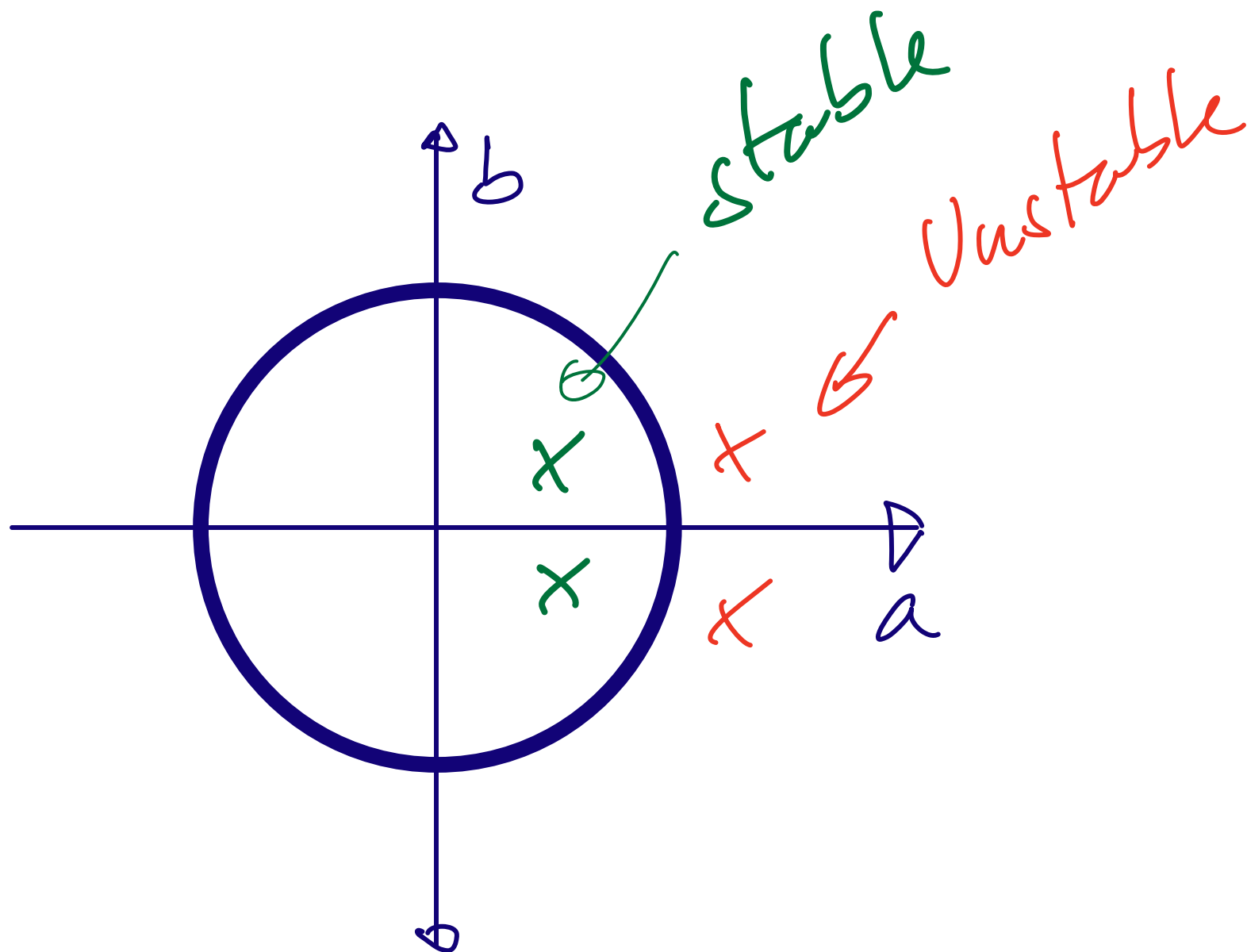
$$y[n] = bx[n - 1] + ay[n - 1] \Rightarrow y = bxz^{-1} + ayz^{-1}$$

$$H(z) = \frac{b}{z - a}$$

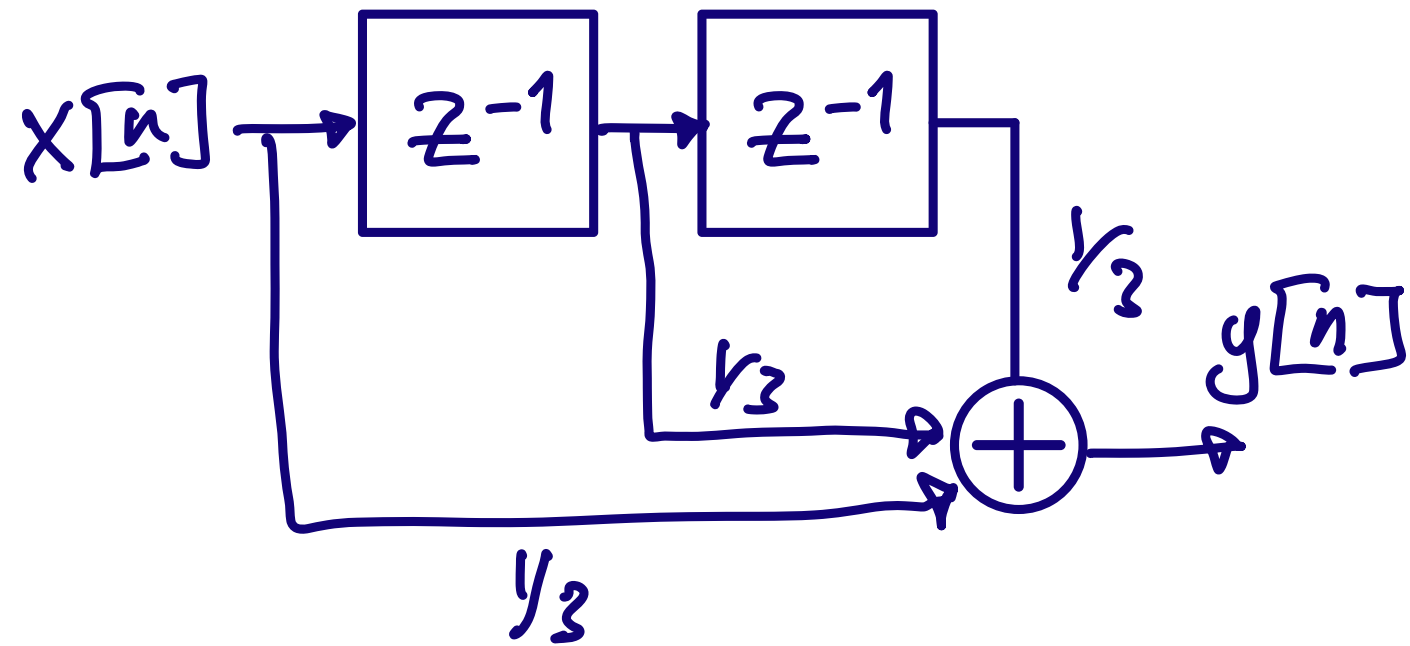
Infinite-impulse response (IIR)

$$h[n] = \begin{cases} k & \text{if } n < 1 \\ a^{n-1}b + a^n k & \text{if } n \geq 1 \end{cases}$$

Head's up: Fig 13.12 in AIC is wrong



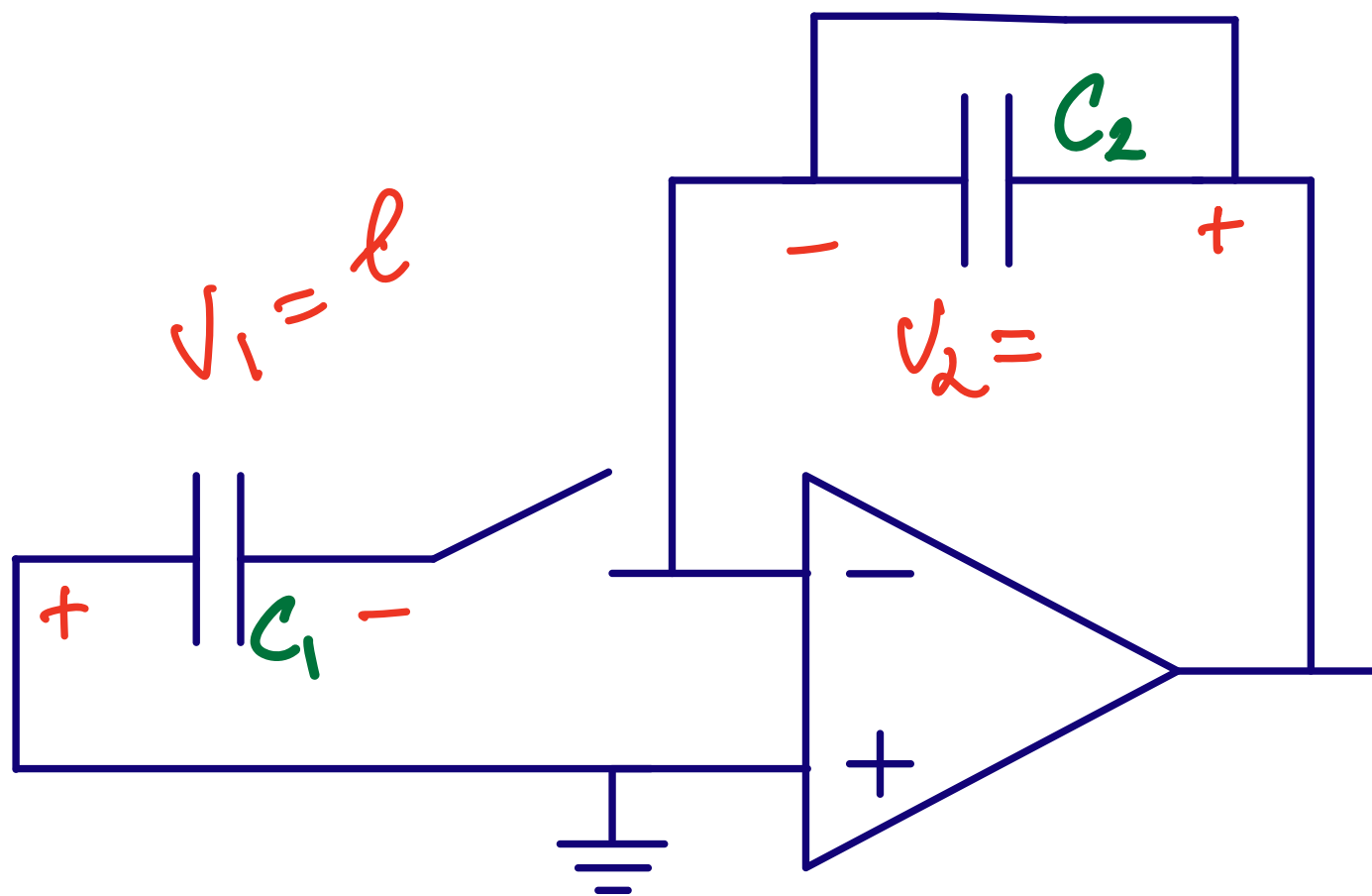
# Finite-impulse response(FIR)



$$H(z) = \frac{1}{3} \sum_{i=0}^2 z^{-i}$$

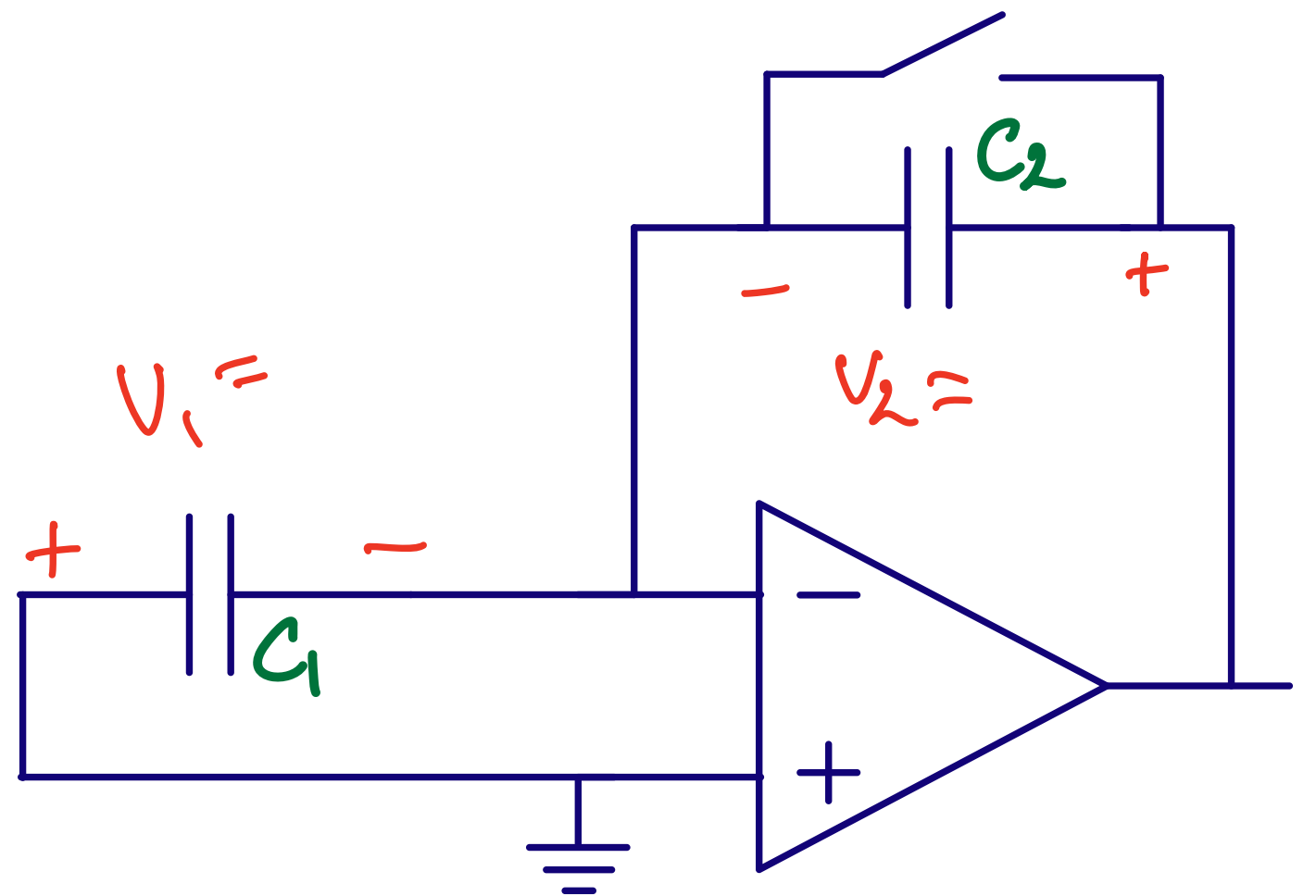
# Switched-Capacitor

**Q: How do switched capacitor circuits work?**



$$Q_1 =$$

$$Q_2 =$$



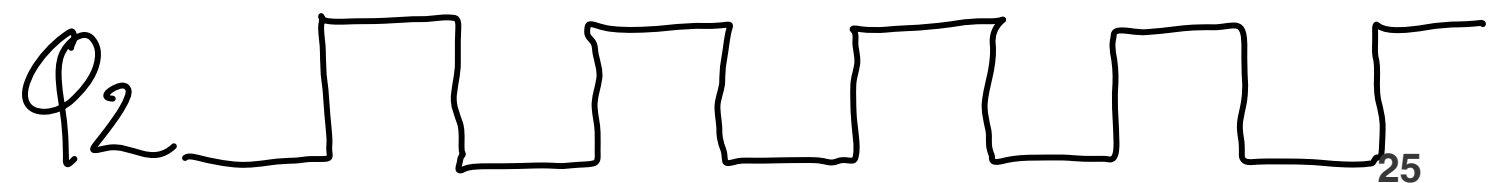
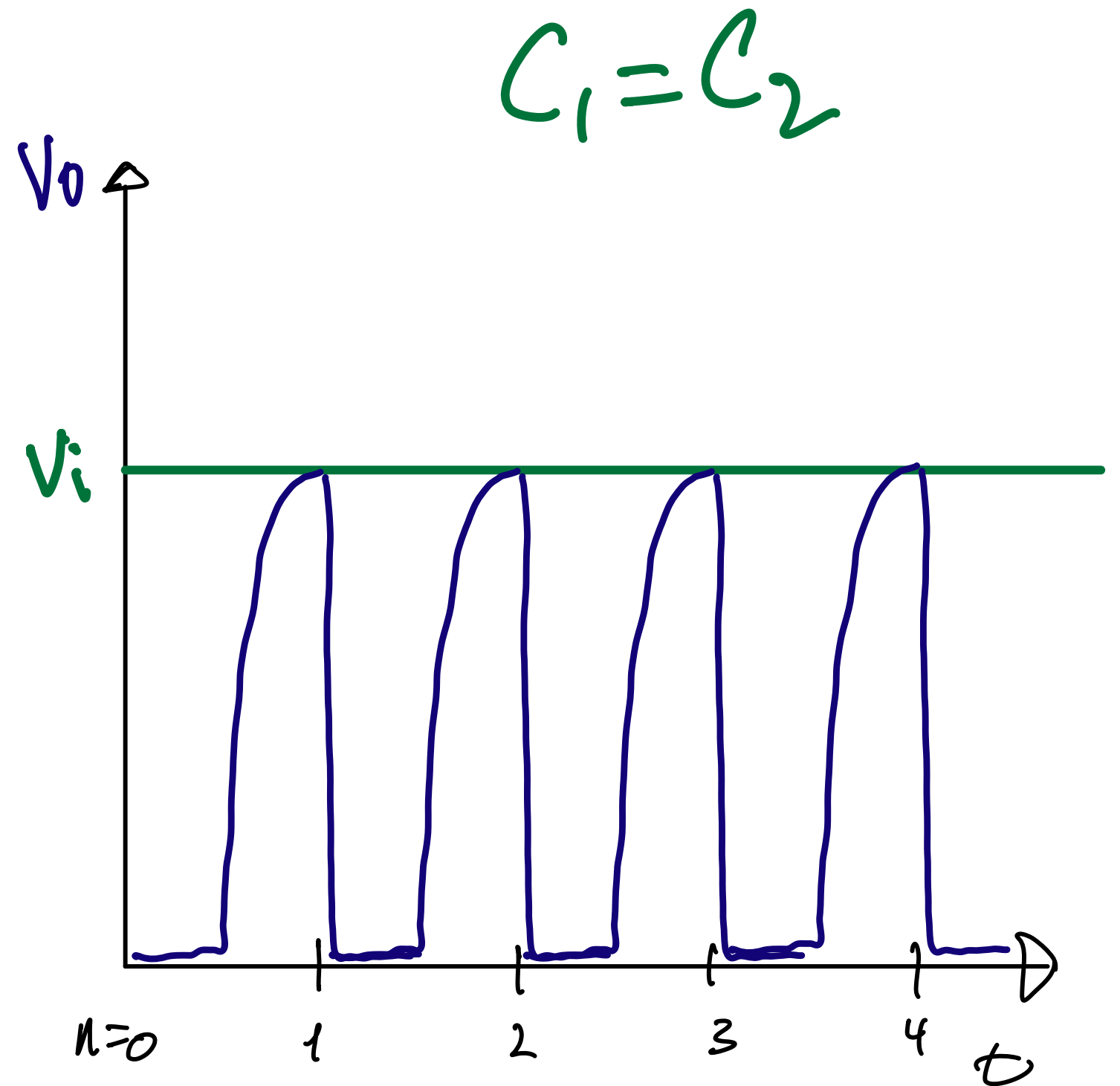
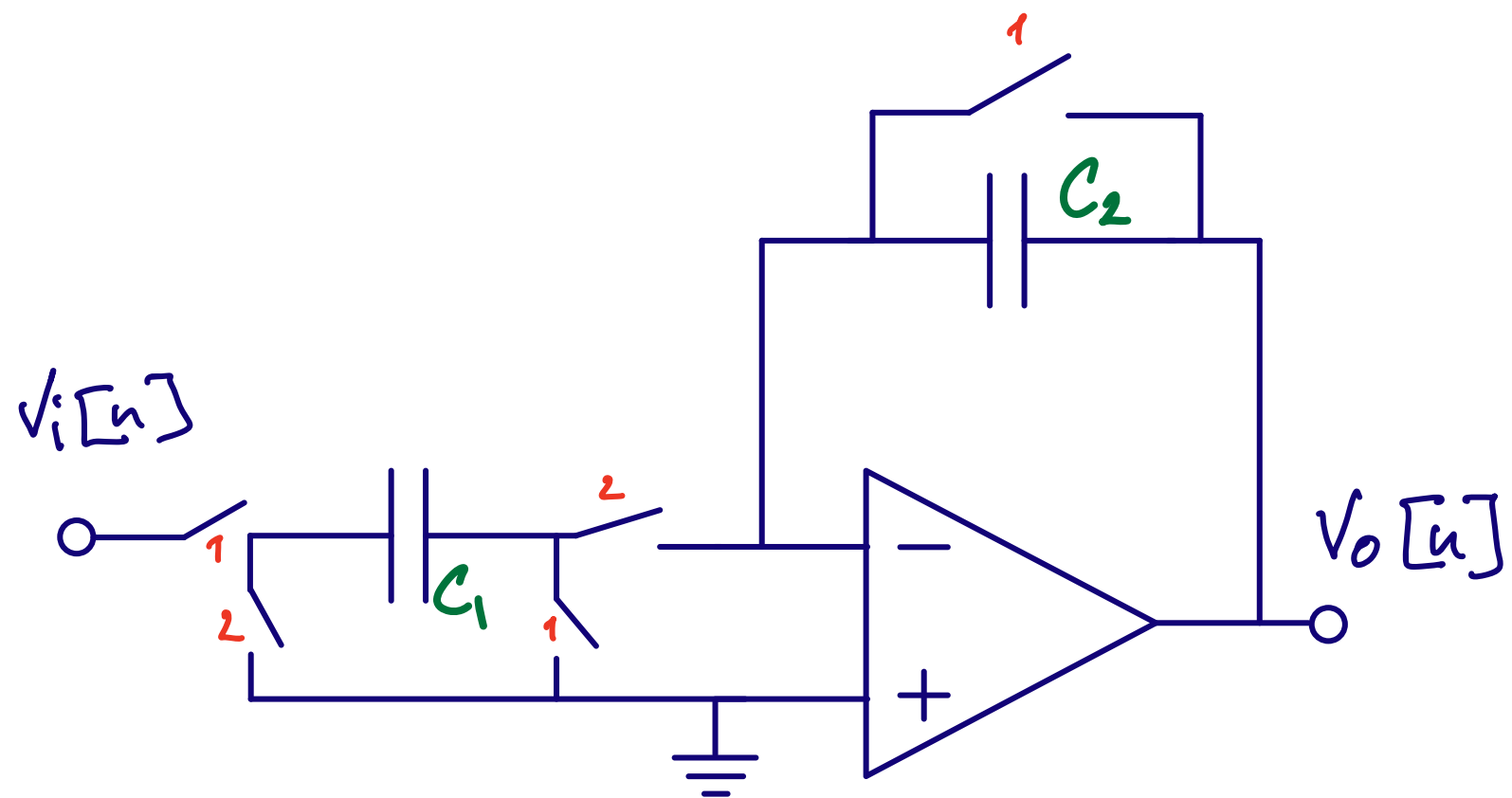
$$Q_1 =$$

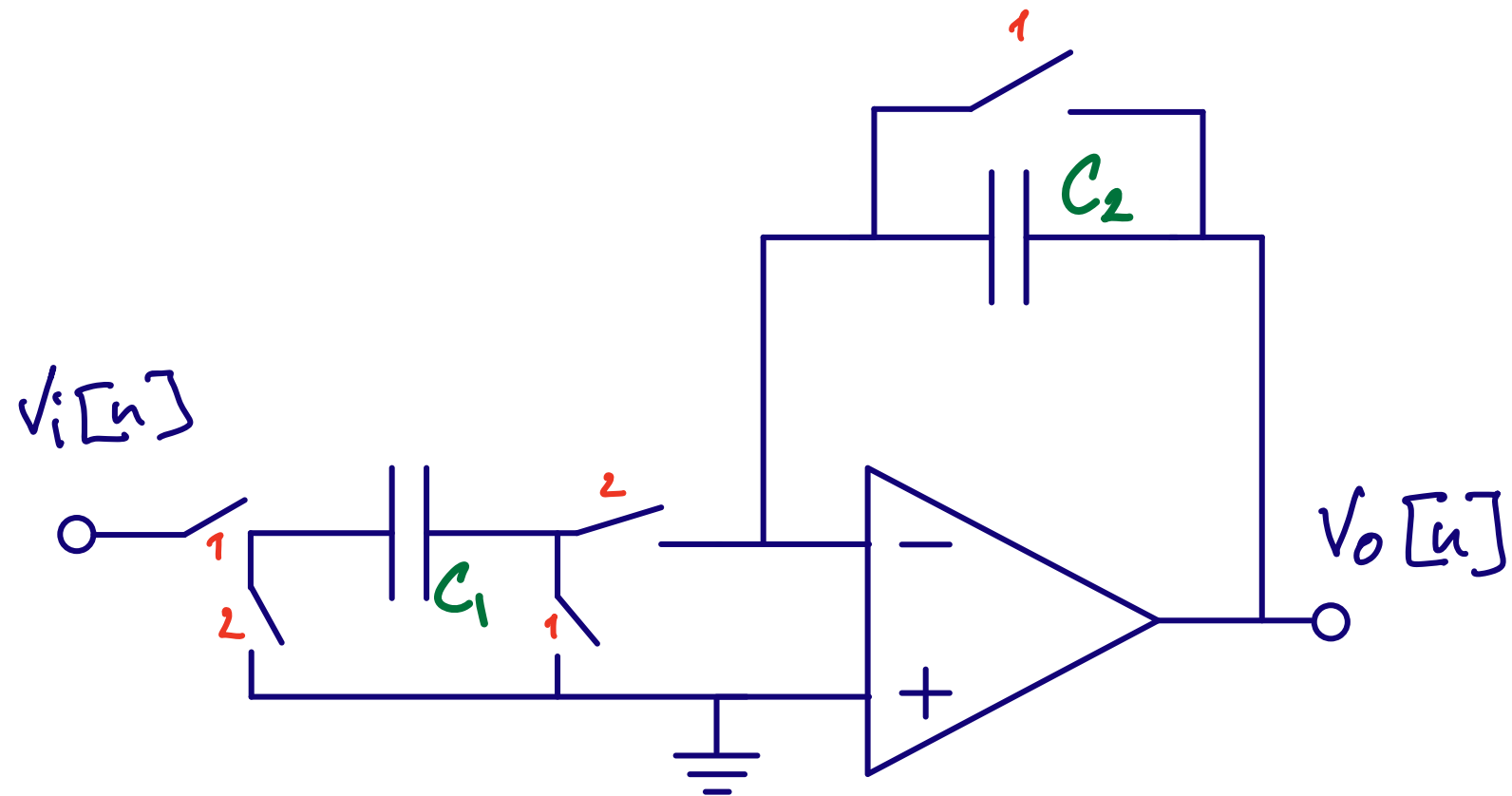
$$Q_2 =$$

# Principles

# Switched capacitor gain circuit





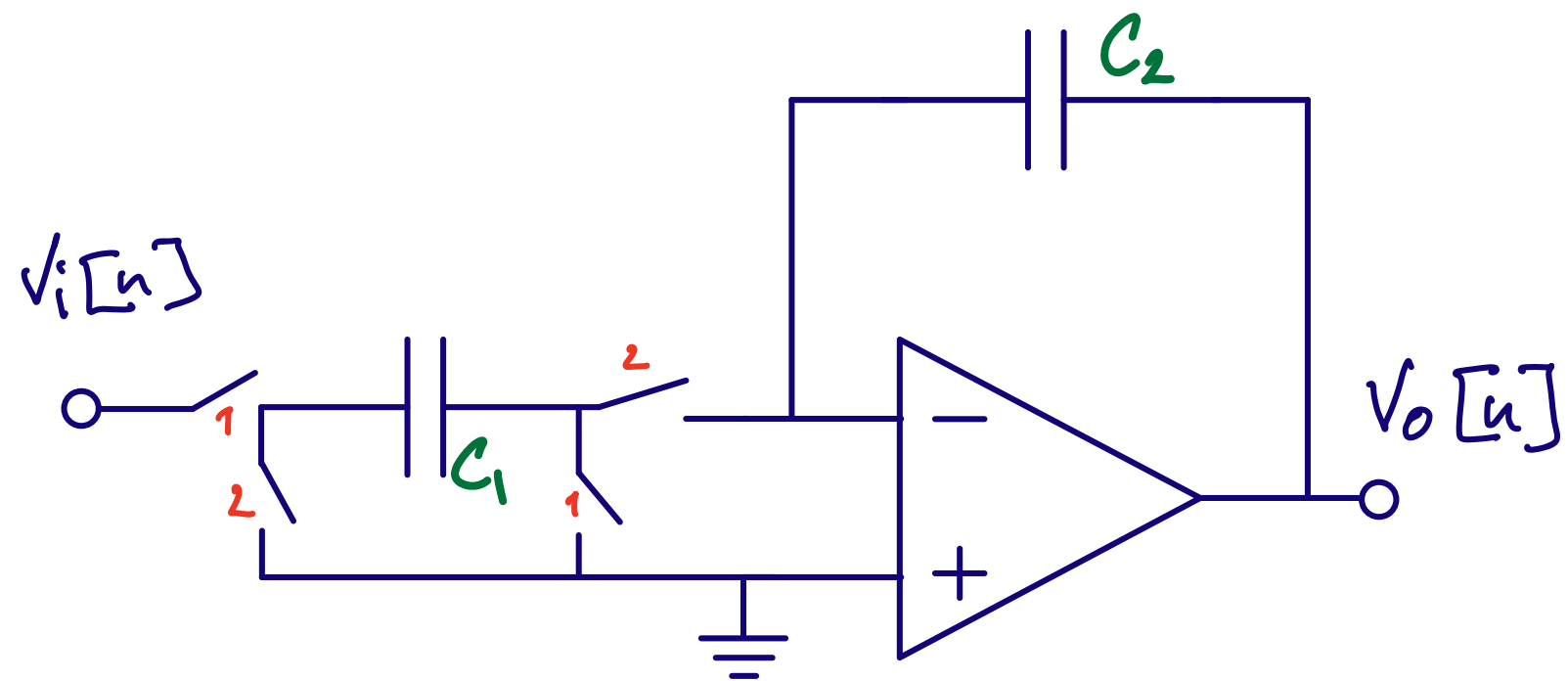


$$V_o[n + 1] = \frac{C_1}{C_2} V_i[n]$$

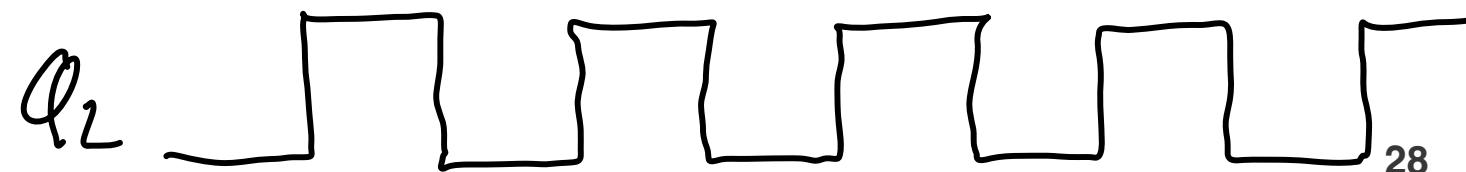
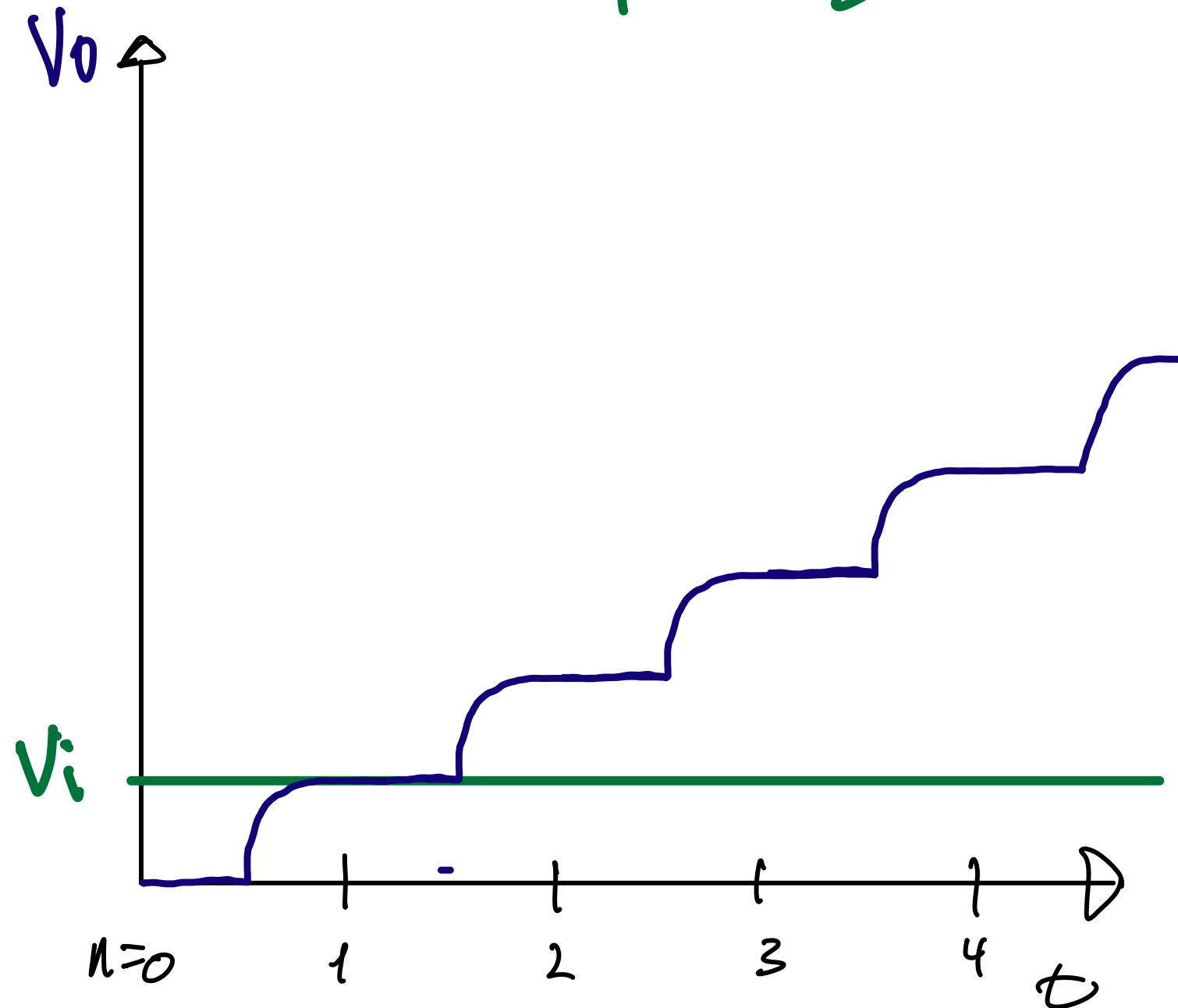
$$V_o z = \frac{C_1}{C_2} V_i$$

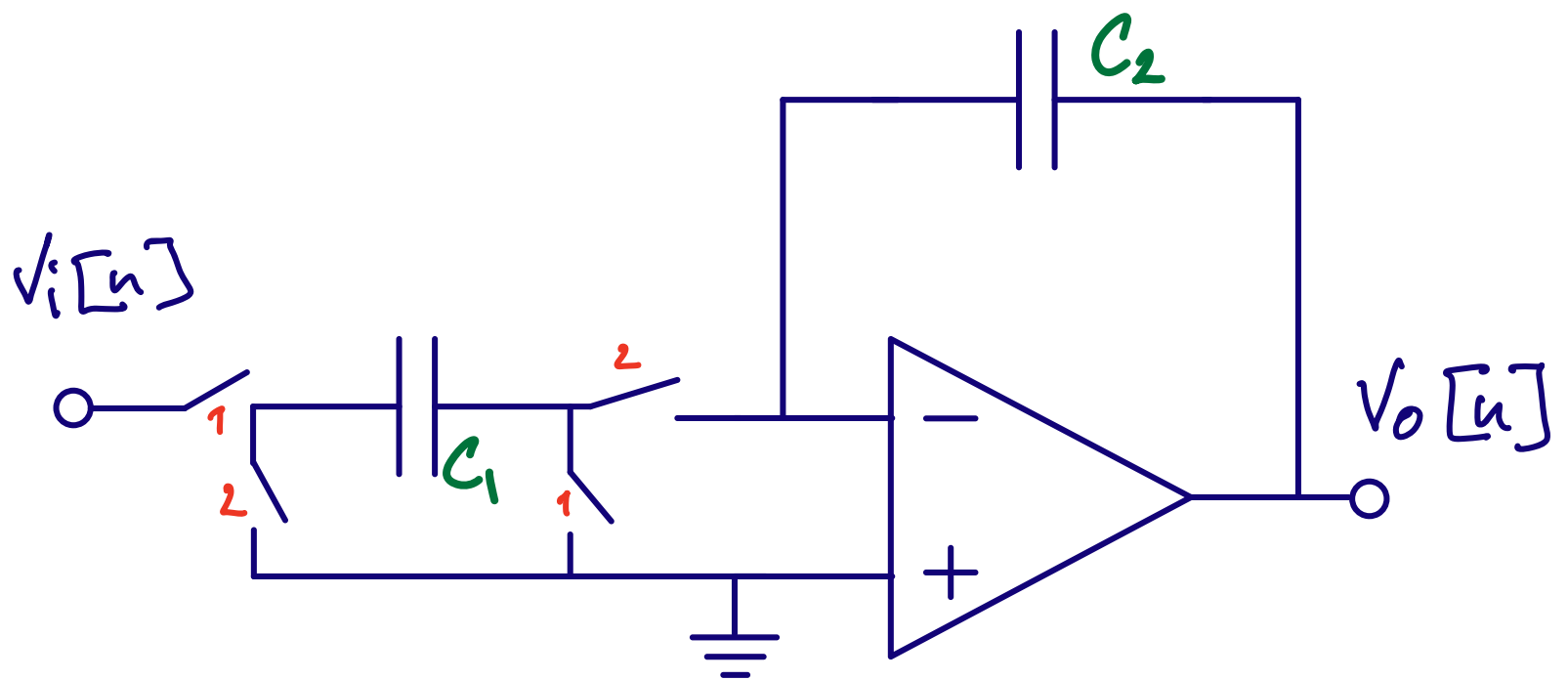
$$\frac{V_o}{V_i} = H(z) = \frac{C_1}{C_2} z^{-1}$$

# Switched capacitor integrator



$$C_1 = C_2$$





$$V_o[n] = V_o[n - 1] + \frac{C_1}{C_2} V_i[n - 1]$$

$$V_o - z^{-1} V_o = \frac{C_1}{C_2} z^{-1} V_i$$

$$H(z) = \frac{C_1}{C_2} \frac{z^{-1}}{z^{-1} + 1} = \frac{C_1}{C_2} \frac{1}{z - 1}$$

Both phases add noise,  $V_n^2 > \frac{2kT}{C}$

Mean

$$\overline{x(t)} = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{+T/2} x(t) dt$$

Mean Square

$$\overline{x^2(t)} = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{+T/2} x^2(t) dt$$

Variance

$$\sigma^2 = \overline{x^2(t)} - \overline{x(t)}^2$$

where  $\sigma$  is the standard deviation.

If mean is removed, or is zero, then

$$\sigma^2 = \overline{x^2(t)}$$

Assume two random processes,  $x_1(t)$  and  $x_2(t)$  with mean of zero (or removed).

$$x_{tot}(t) = x_1(t) + x_2(t)$$

$$x_{tot}^2(t) = x_1^2(t) + x_2^2(t) + 2x_1(t)x_2(t)$$

Variance (assuming mean of zero)

$$\sigma_{tot}^2 = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{+T/2} x_{tot}^2(t) dt$$

$$\sigma_{tot}^2 = \sigma_1^2 + \sigma_2^2 + \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{+T/2} 2x_1(t)x_2(t) dt$$

**Assuming uncorrelated processes**

**(covariance is zero), then**

$$\sigma_{tot}^2 = \sigma_1^2 + \sigma_2^2$$

# Sub-circuits for SC-circuits

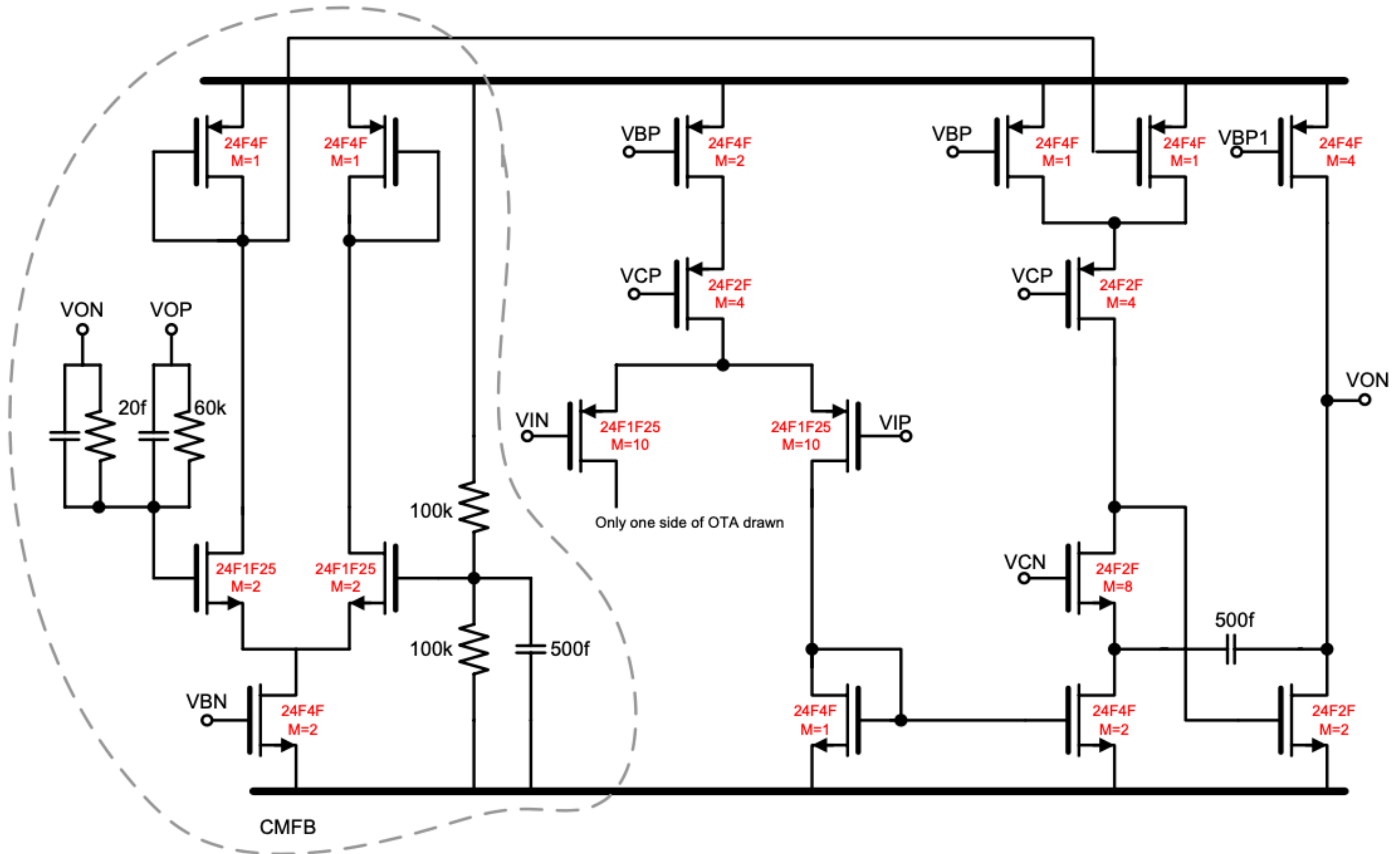


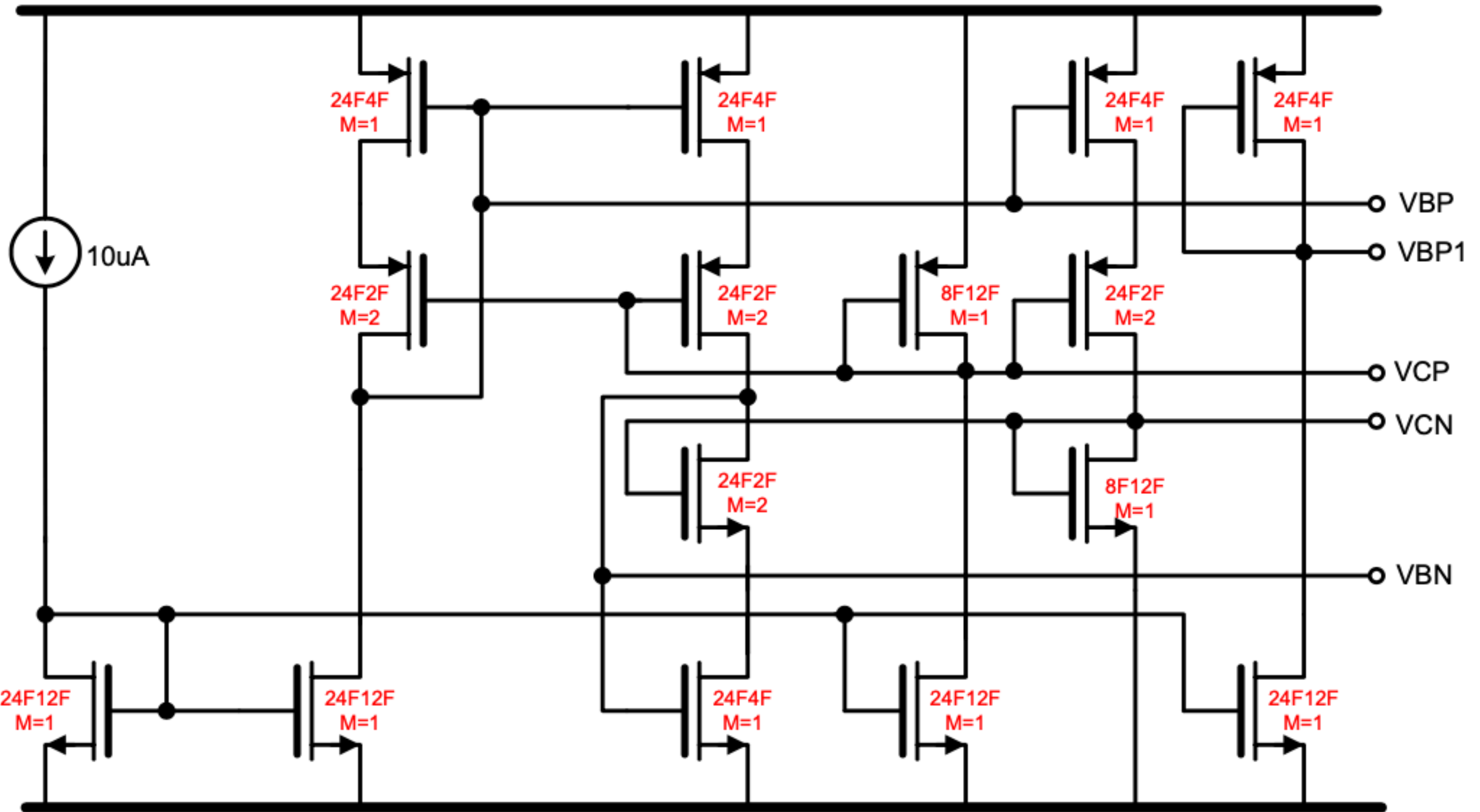
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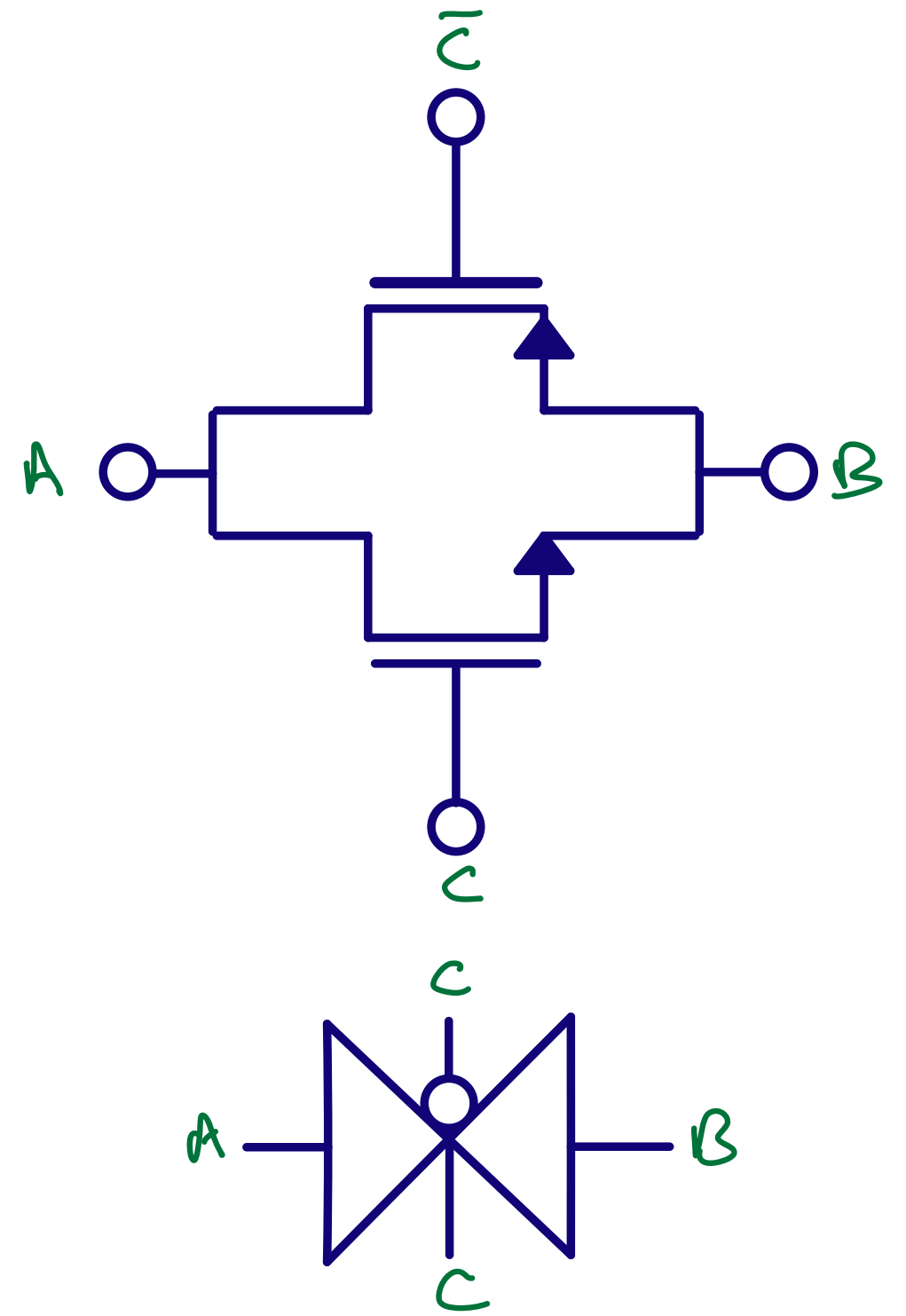
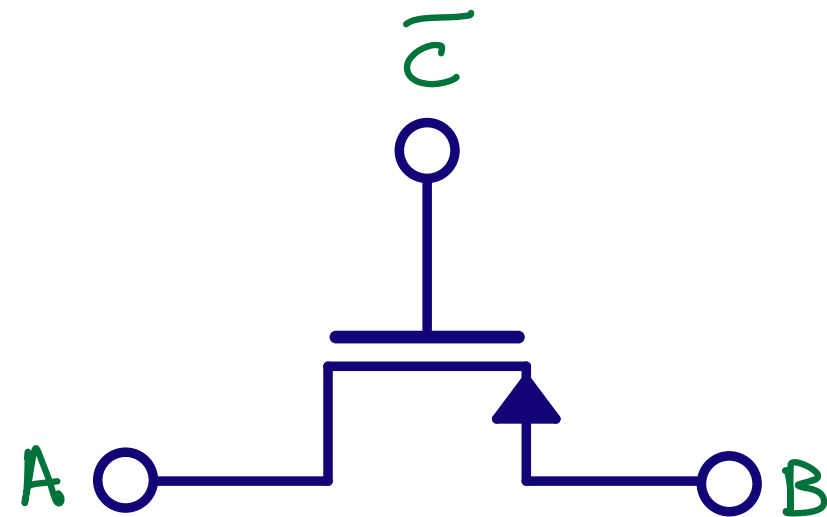
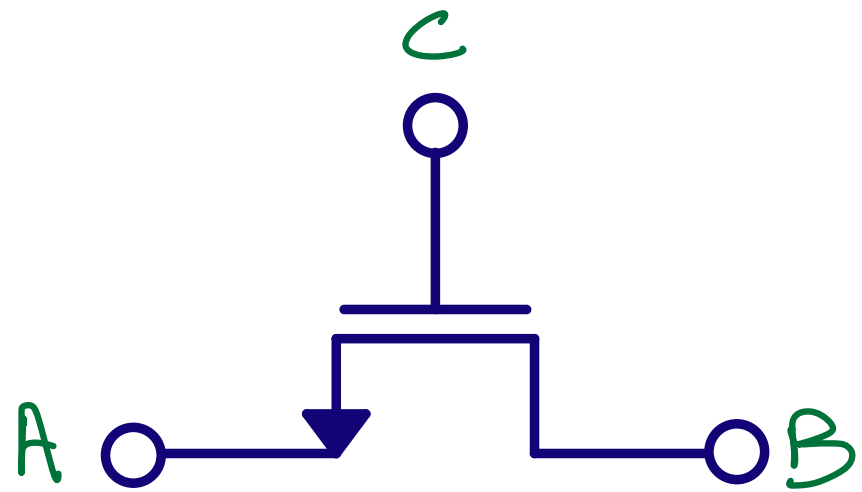
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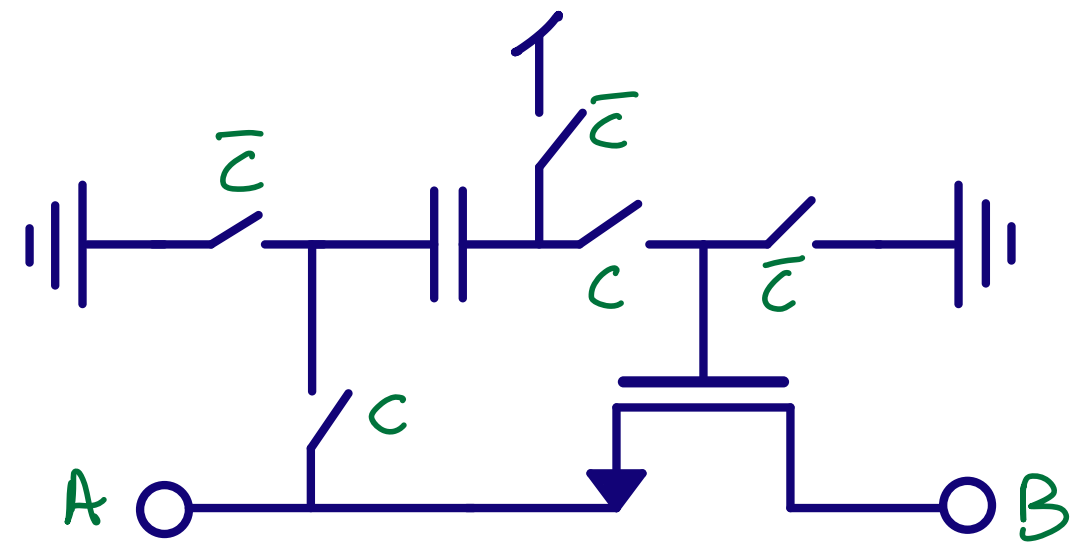
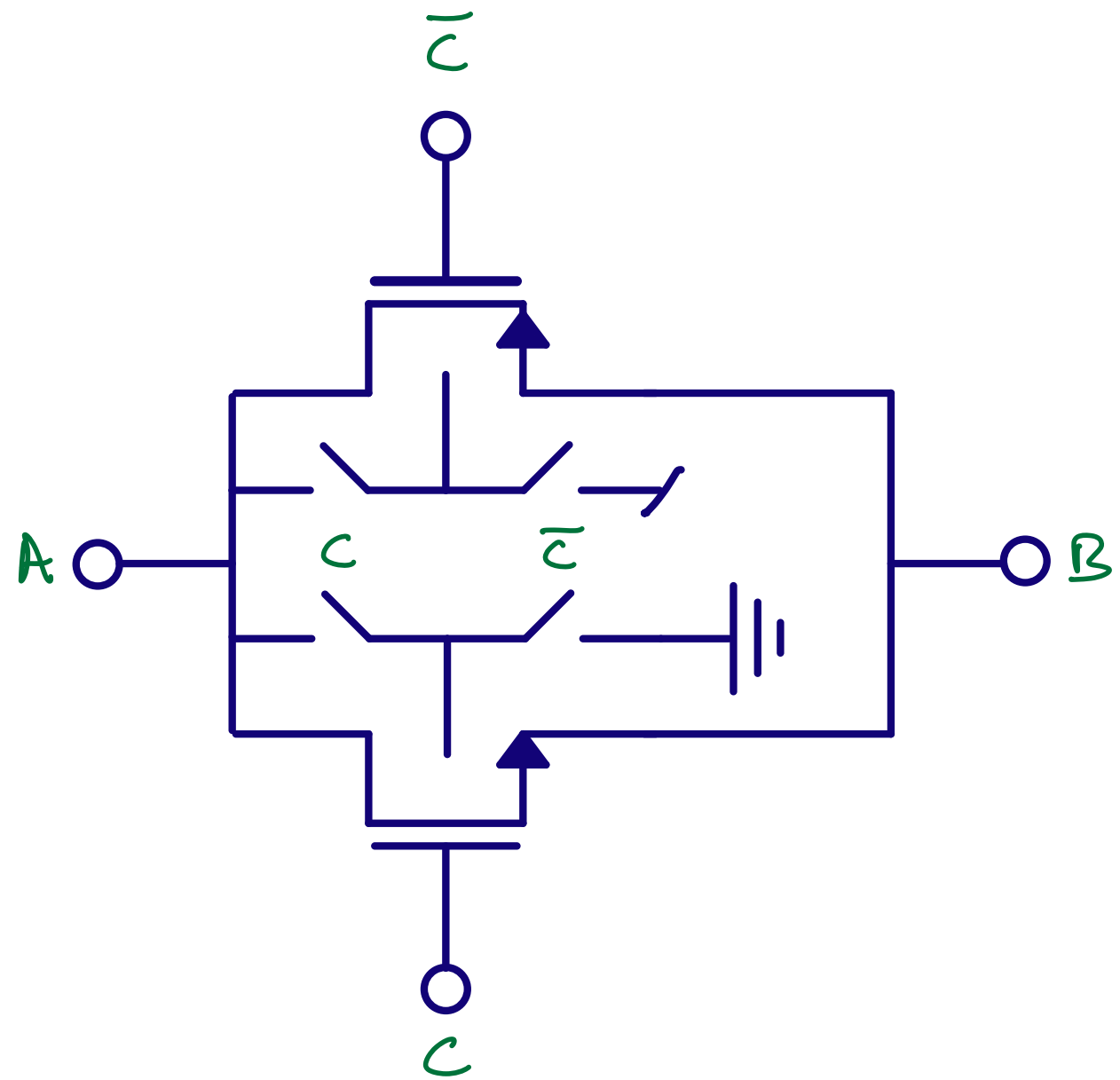
**Q: How many different OTAs do you know?**

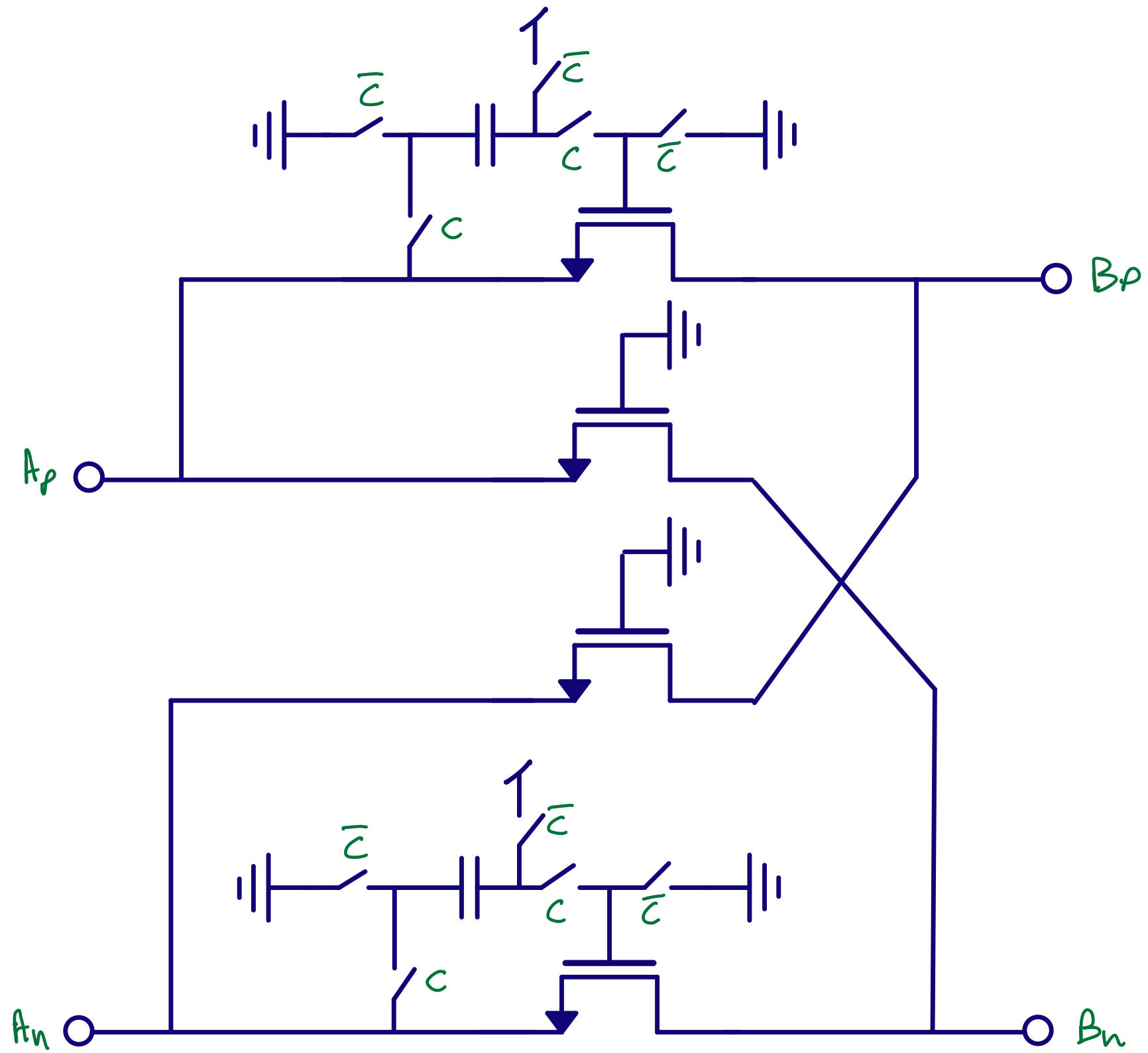




# Switches

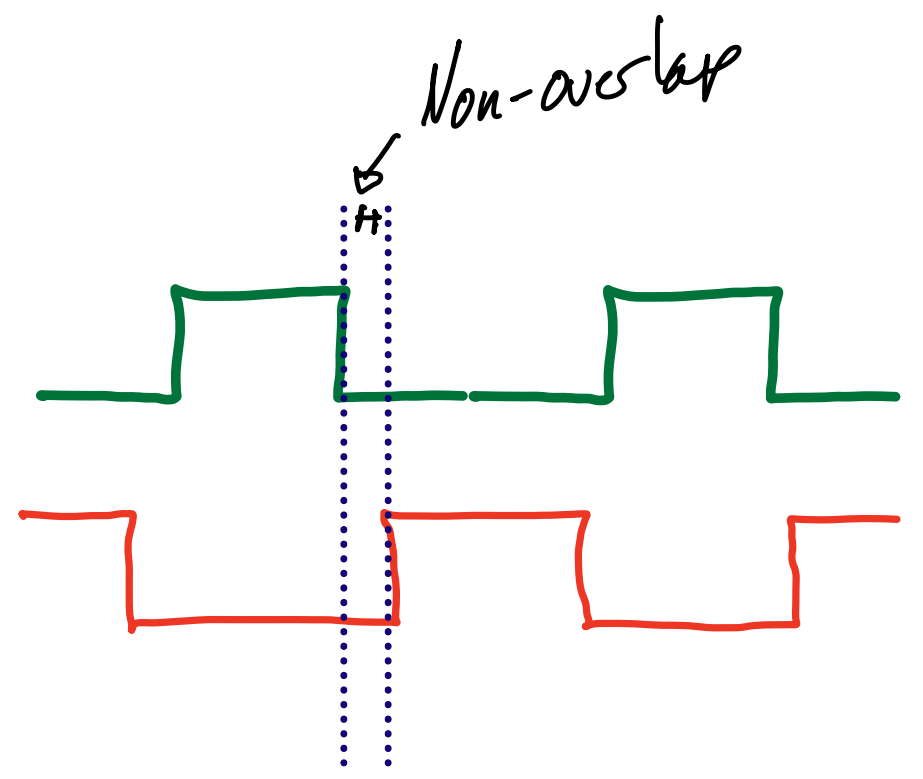
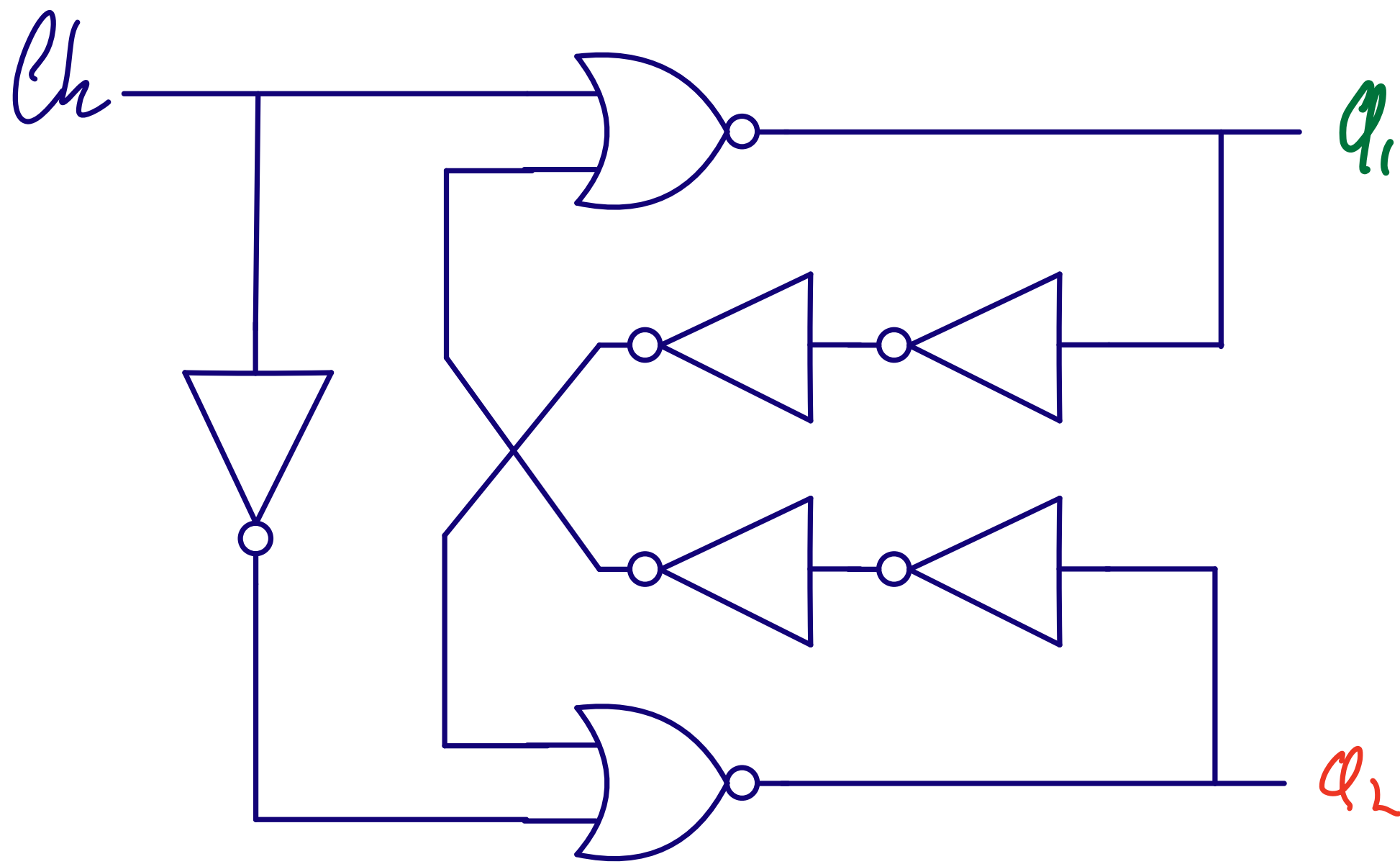






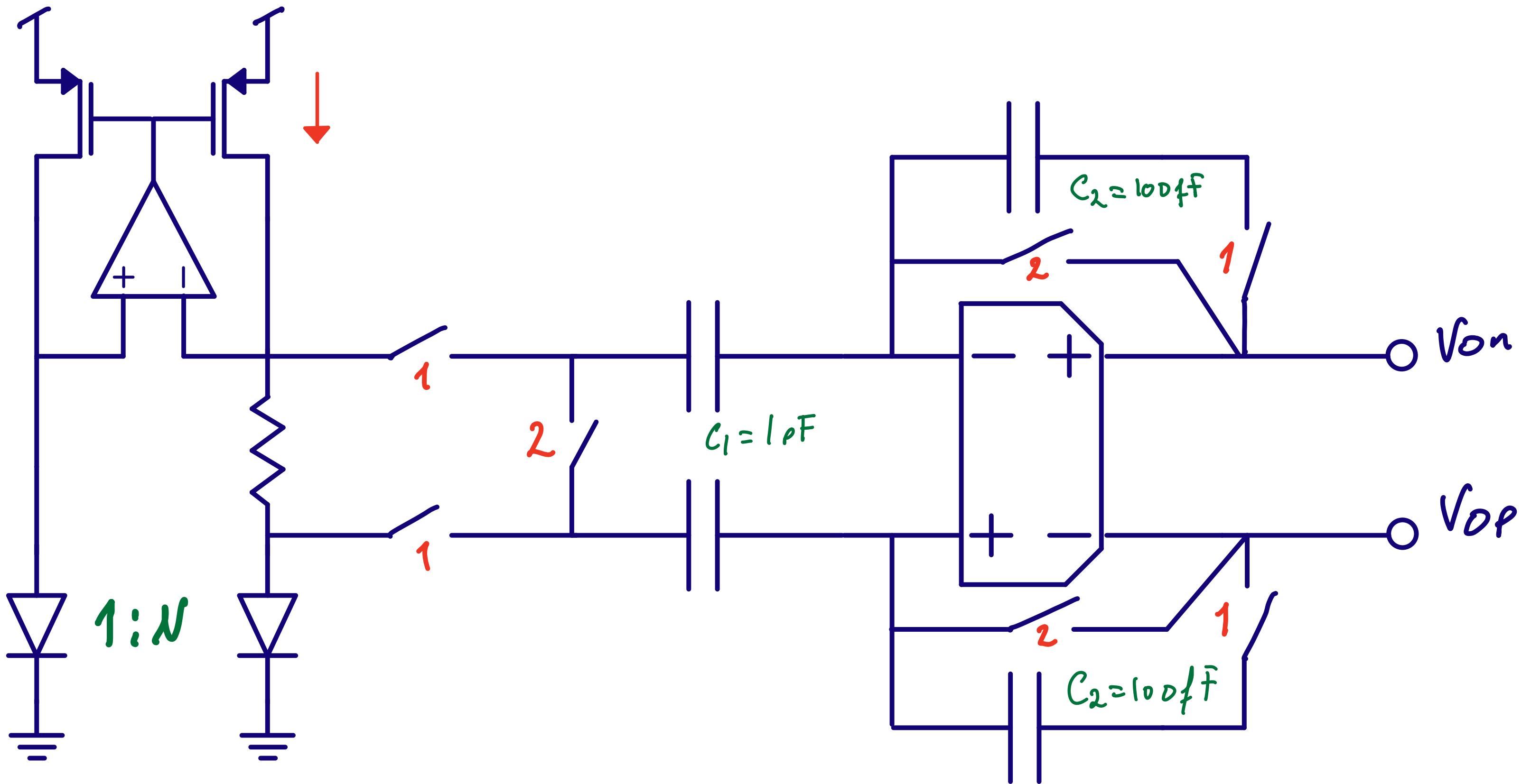


# Non-overlapping clocks



Example

**Q: What is the gain?**



# In the book that you really should read

Parasitic sensitive integrator

Gain circuit that zero's OTA offset

Correlated double sampling

VCO

Peak detectors

**Thanks!**

