

TFE4188 - Lecture 4

Analog frontend and filters

Why

The world is analog and is written in the mathematics of calculus ¹

$$\oint_{\partial\Omega} \mathbf{E} \cdot d\mathbf{S} = \frac{1}{\epsilon_0} \iiint_V \rho \cdot dV$$

Relates net electric flux to net enclosed electric charge

$$\oint_{\partial\Omega} \mathbf{B} \cdot d\mathbf{S} = 0$$

Relates net magnetic flux to net enclosed magnetic charge

$$\oint_{\partial\Sigma} \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt} \iint_{\Sigma} \mathbf{B} \cdot d\mathbf{S}$$


Relates induced electric field to changing magnetic flux

$$\oint_{\partial\Sigma} \mathbf{B} \cdot d\mathbf{l} = \mu_0 \left(\iint_{\Sigma} \mathbf{J} \cdot d\mathbf{S} + \epsilon_0 \frac{d}{dt} \iint_{\Sigma} \mathbf{E} \cdot d\mathbf{S} \right)$$

Relates induced magnetic field to changing electric flux and to current

¹ [Maxwell's equations](#)





The behavior of particles is written in the mathematics of quantum mechanics

$$\psi(x, t) = Ae^{j(kx - \omega t)}$$

Probability amplitude of a particle

$$\frac{1}{2m} \frac{\hbar}{j^2} \frac{\partial^2}{\partial x^2} \psi(x, t) + U(x)\psi(x, t) = -\frac{\hbar}{j} \frac{\partial}{\partial t} \psi(x, t)$$

Time evolution of the energy of a particle²

$$\frac{n_n}{n_p} = \frac{e^{(E_p - \mu)/kT} + 1}{e^{(E_n - \mu)/kT} + 1}$$

Relates the average number of fermions in thermal equilibrium to the energy of a single-particle state³

² Schrödinger equation

³ Fermi-Dirac statistics

The abstract digital world is written in the mathematics of boolean algebra⁴

1 = True, 0 = False

A	B	NOT(A AND B)
0	0	1
0	1	1
1	0	1
1	1	0

All digital processing can be made with the NOT(A AND B) function!

⁴ Boolean algebra



People that make
digital circuits can
easily reuse the
work of others



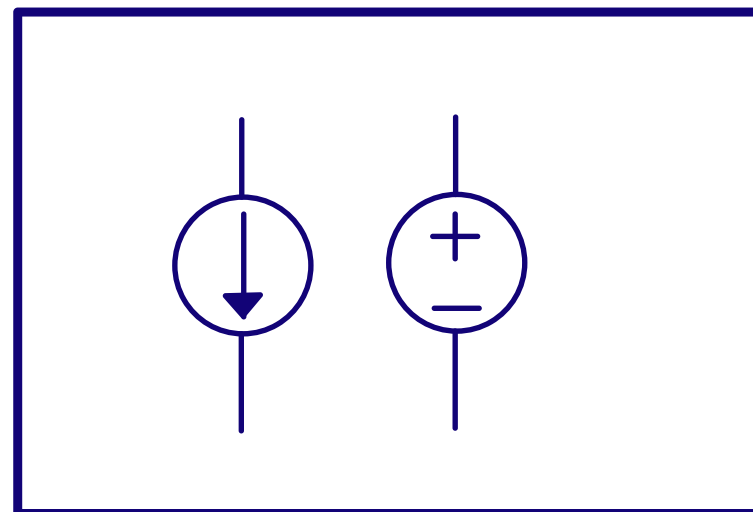


People that make analog circuits can learn from others, but need to deal with the real world on their own

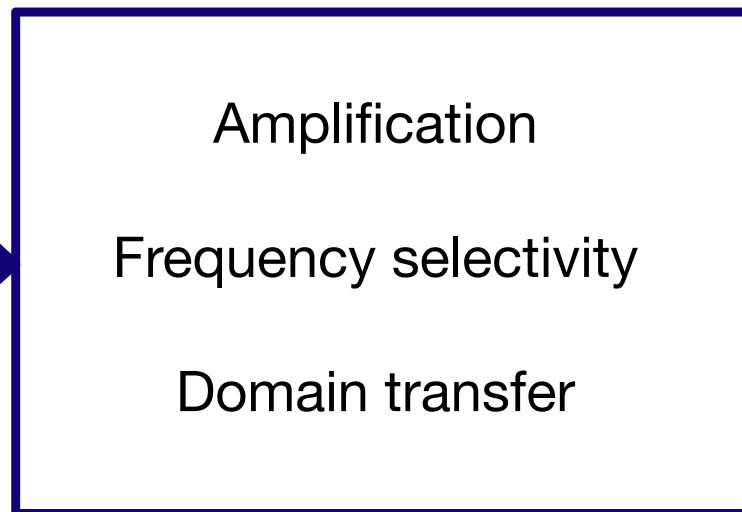
Should we do as much as possible in the abstract digital world?



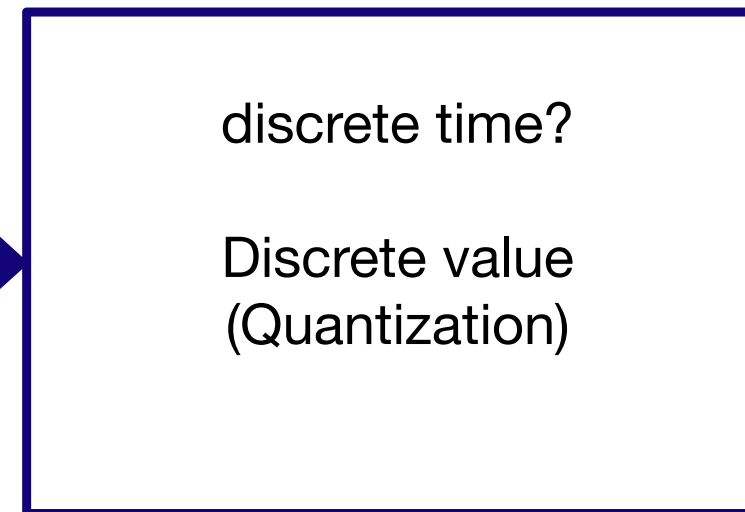
Sensor



AFE

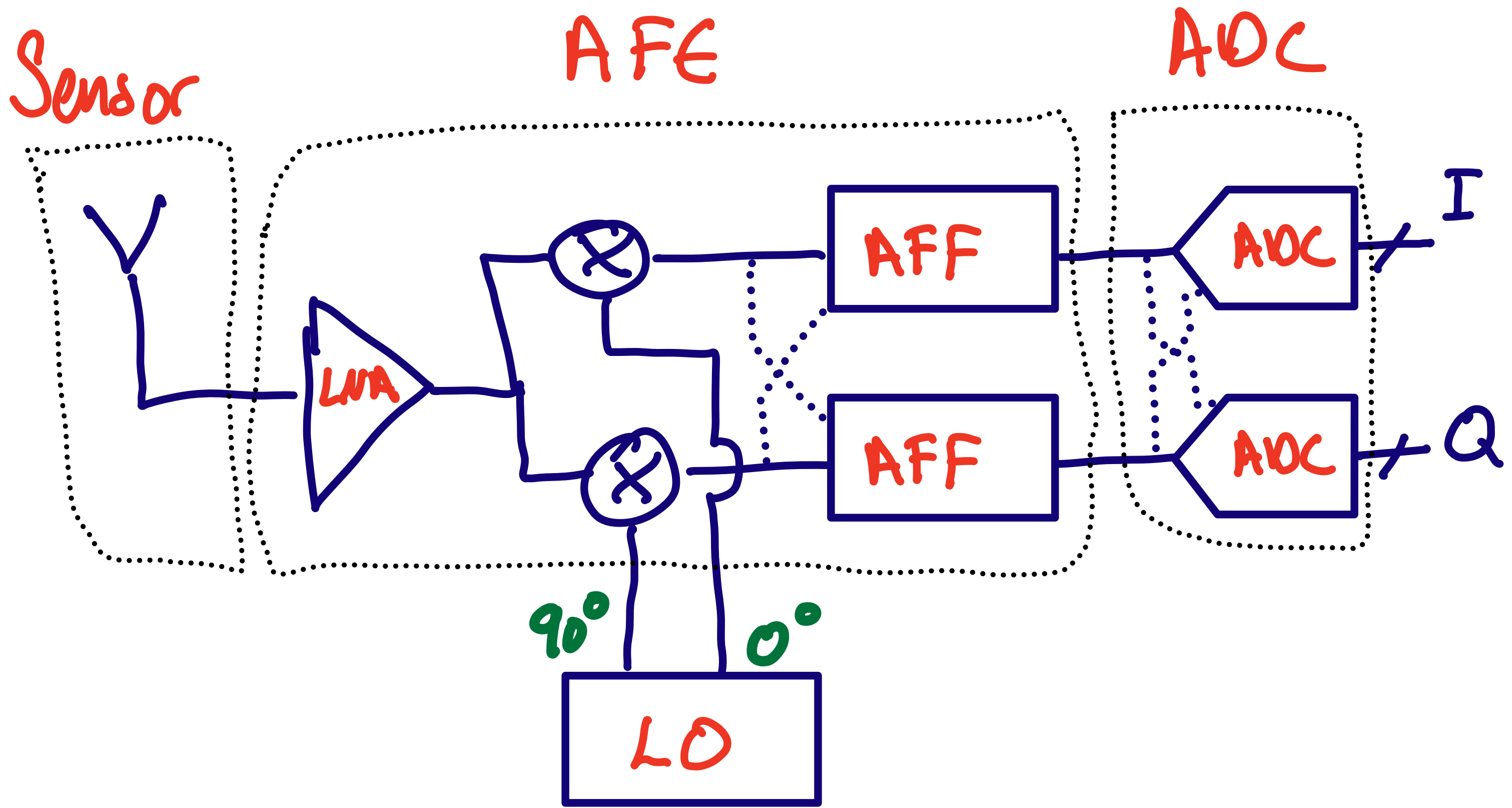


ADC



Bits





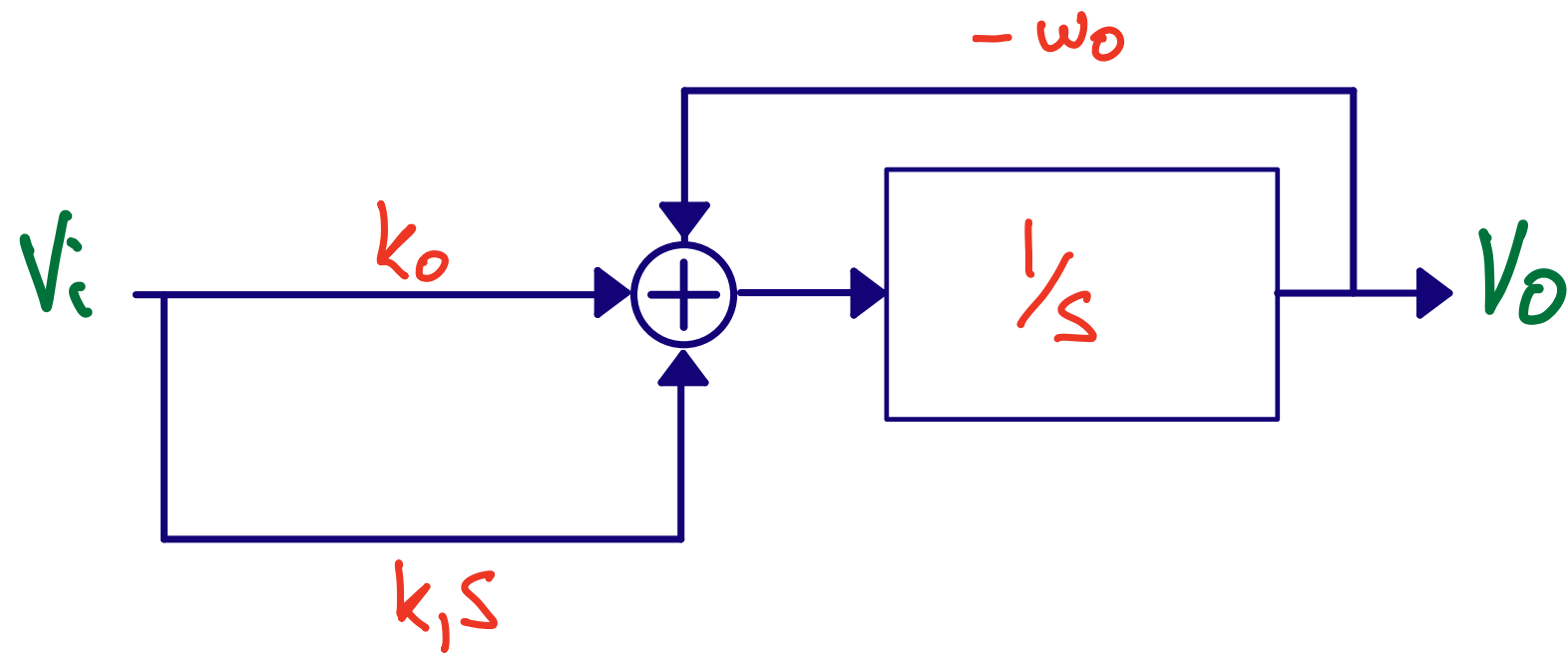
You must know application before you make the AFE!

Filters

A combination of 1'st and 2'nd order stages can synthesize any order filter

First order filter

$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{k_1 s + k_0}{s + \omega_0}$$



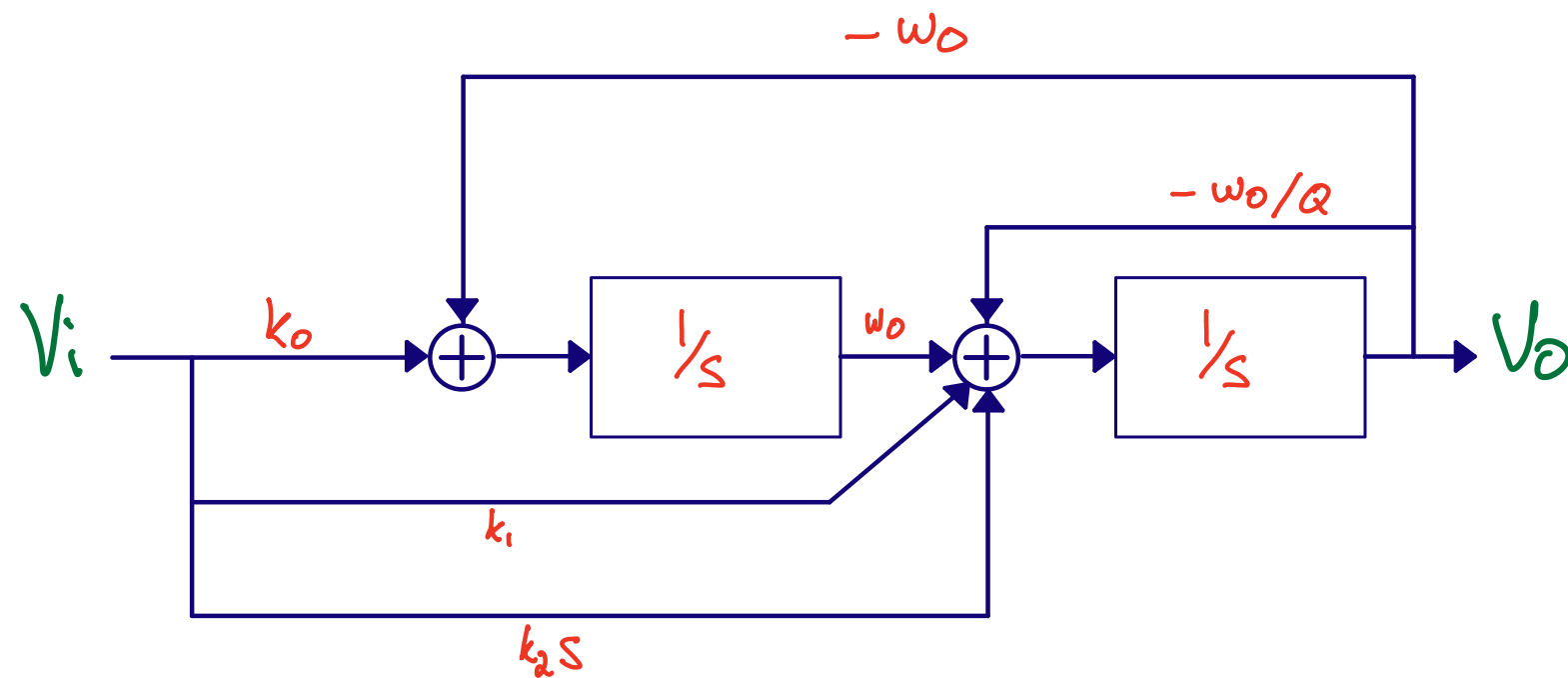
Q: Try to calculate the transfer function from the figure

Second order filter

Bi-quadratic is a general purpose second order filter.

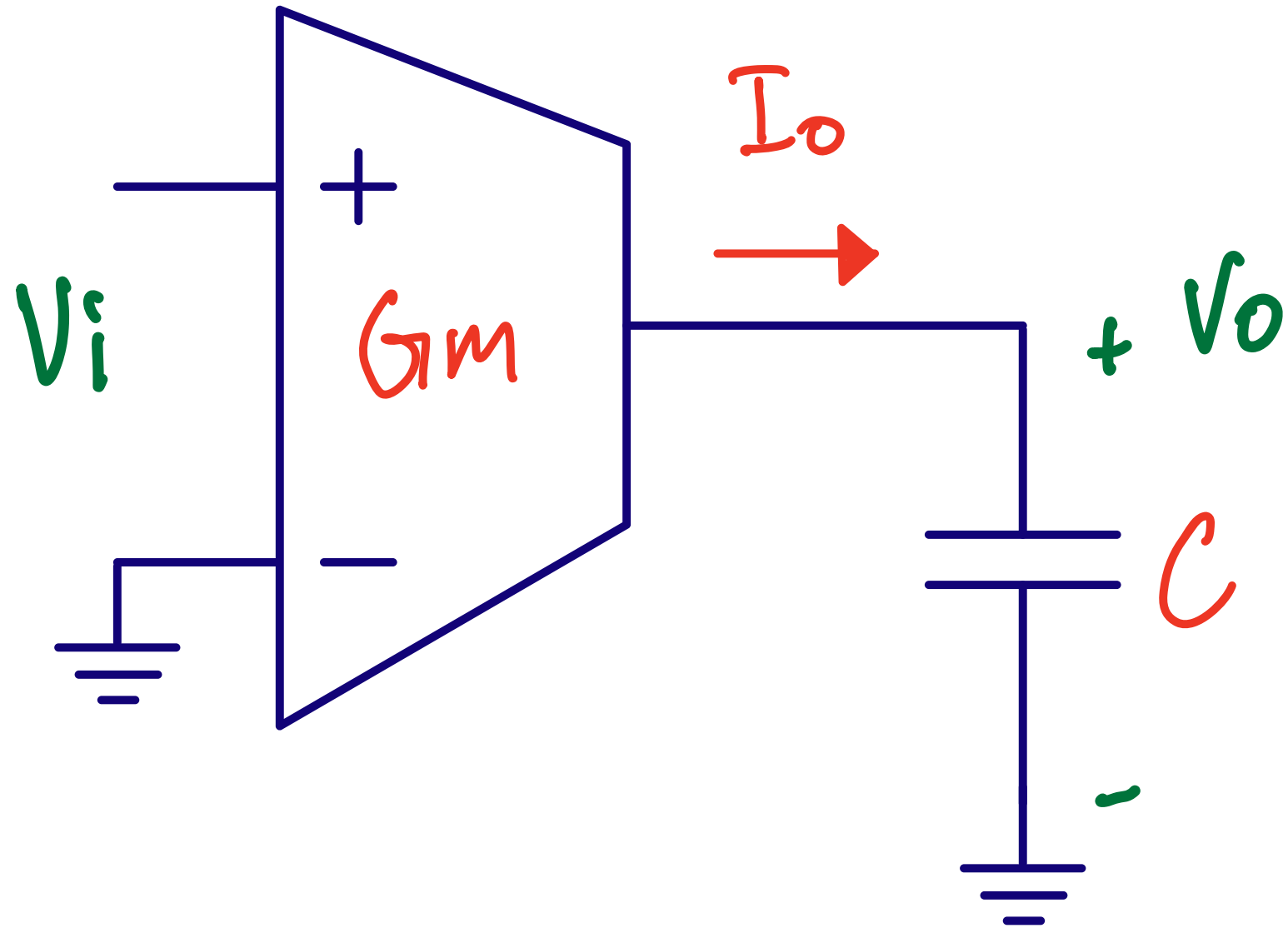
$$H(s) = \frac{k_2 s^2 + k_1 s + k_0}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2}$$

Q: Try to calculate the transfer function from the figure



How do we implement the filter sections?

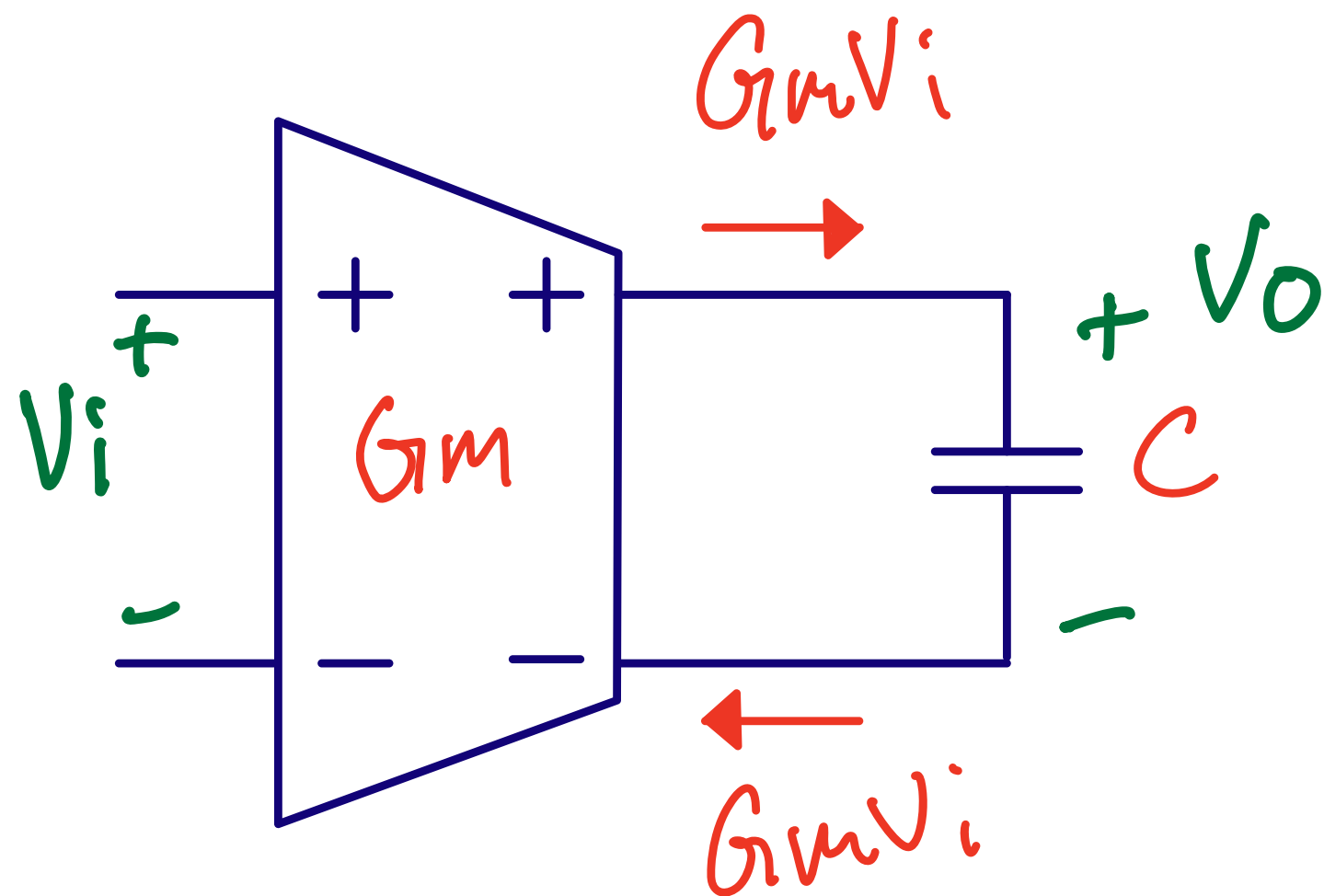
Gmm - C



$$V_o = \frac{I_o}{sC} = \frac{\omega_{ti}}{s} V_i$$

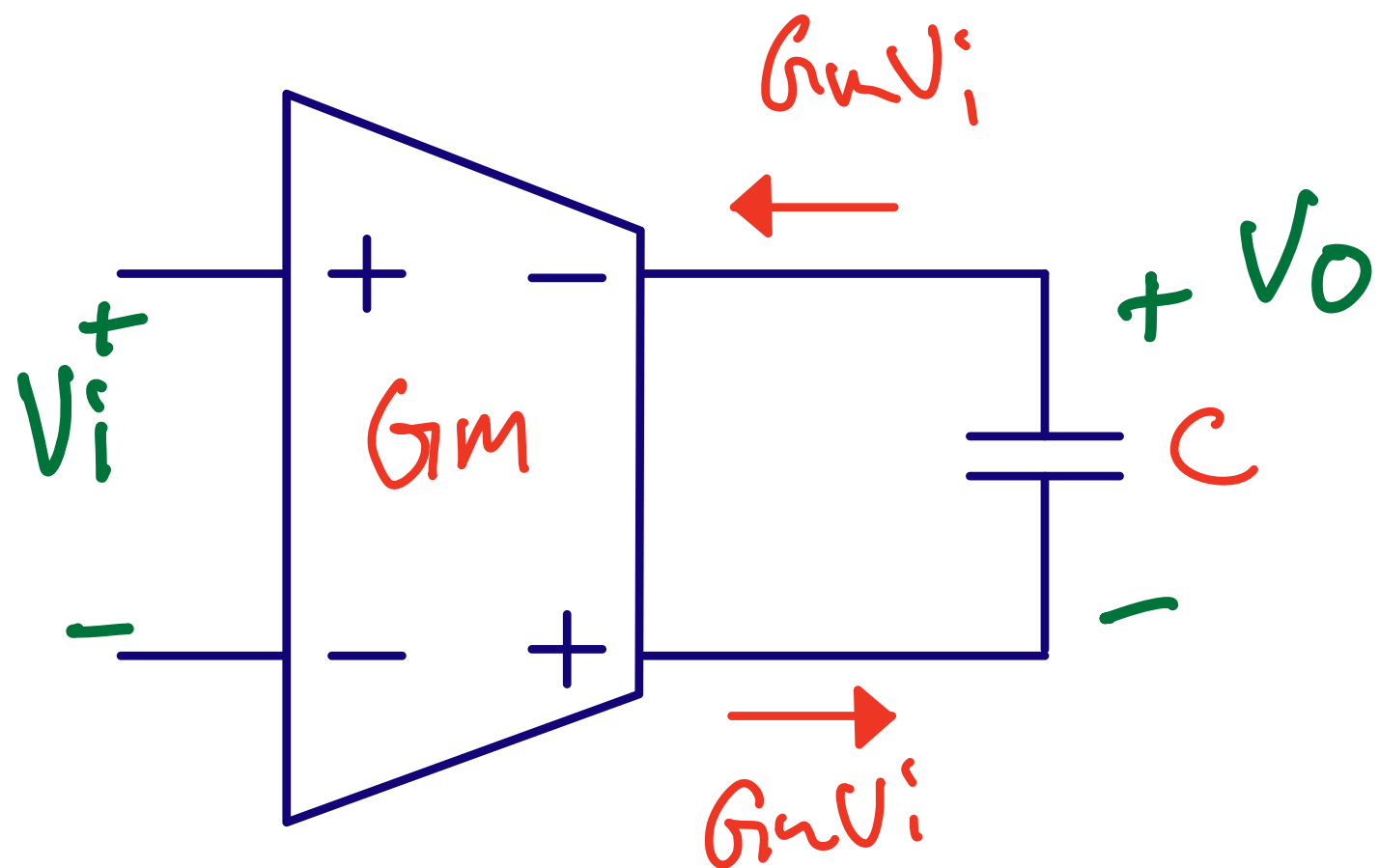
$$\omega_{ti} = \frac{G_m}{C}$$

Q: $g_m = G_m$?

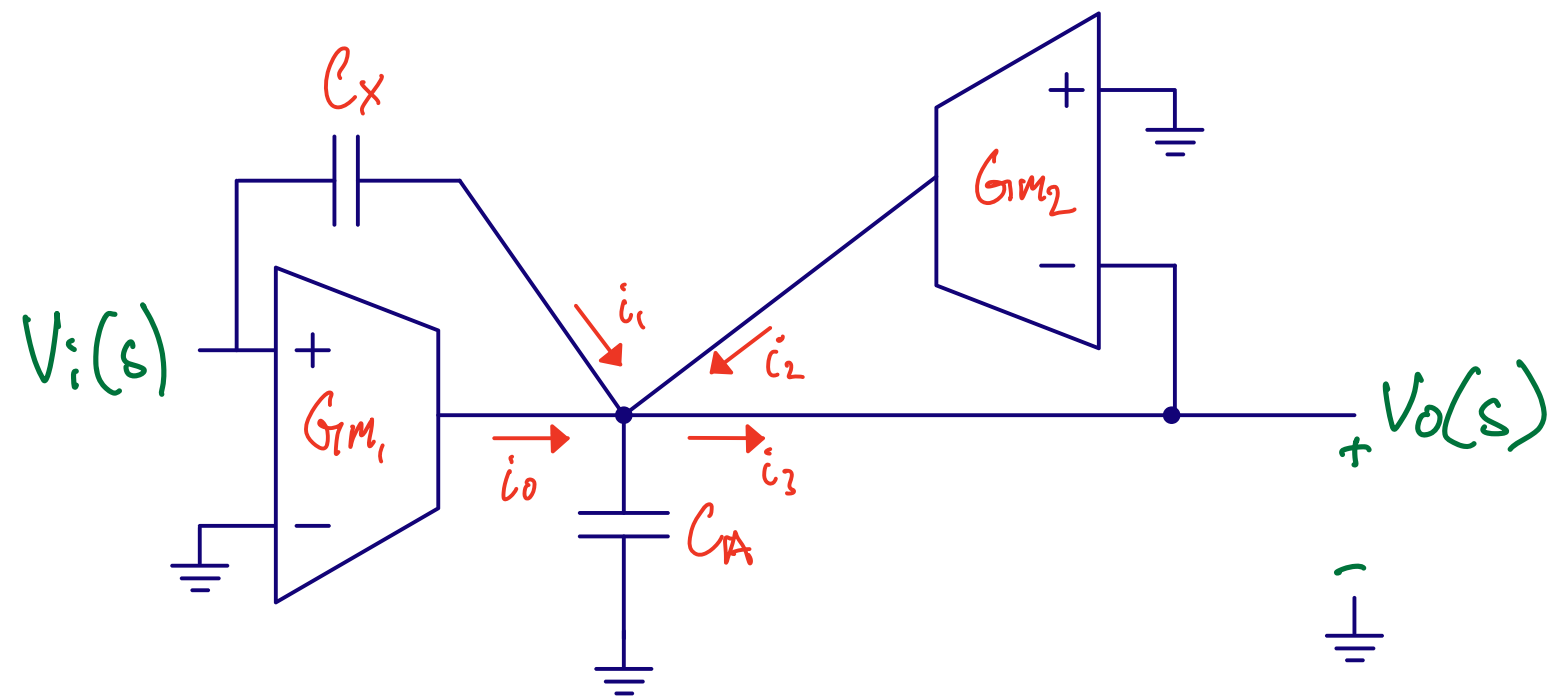


$$sCV_o = G_m V_i$$

$$H(s) = \frac{V_o}{V_i} = \frac{G_m}{sC}$$



Q: Calculate the transfer function



$$i_0 =$$

$$i_1 =$$

$$i_2 =$$

$$i_3 =$$

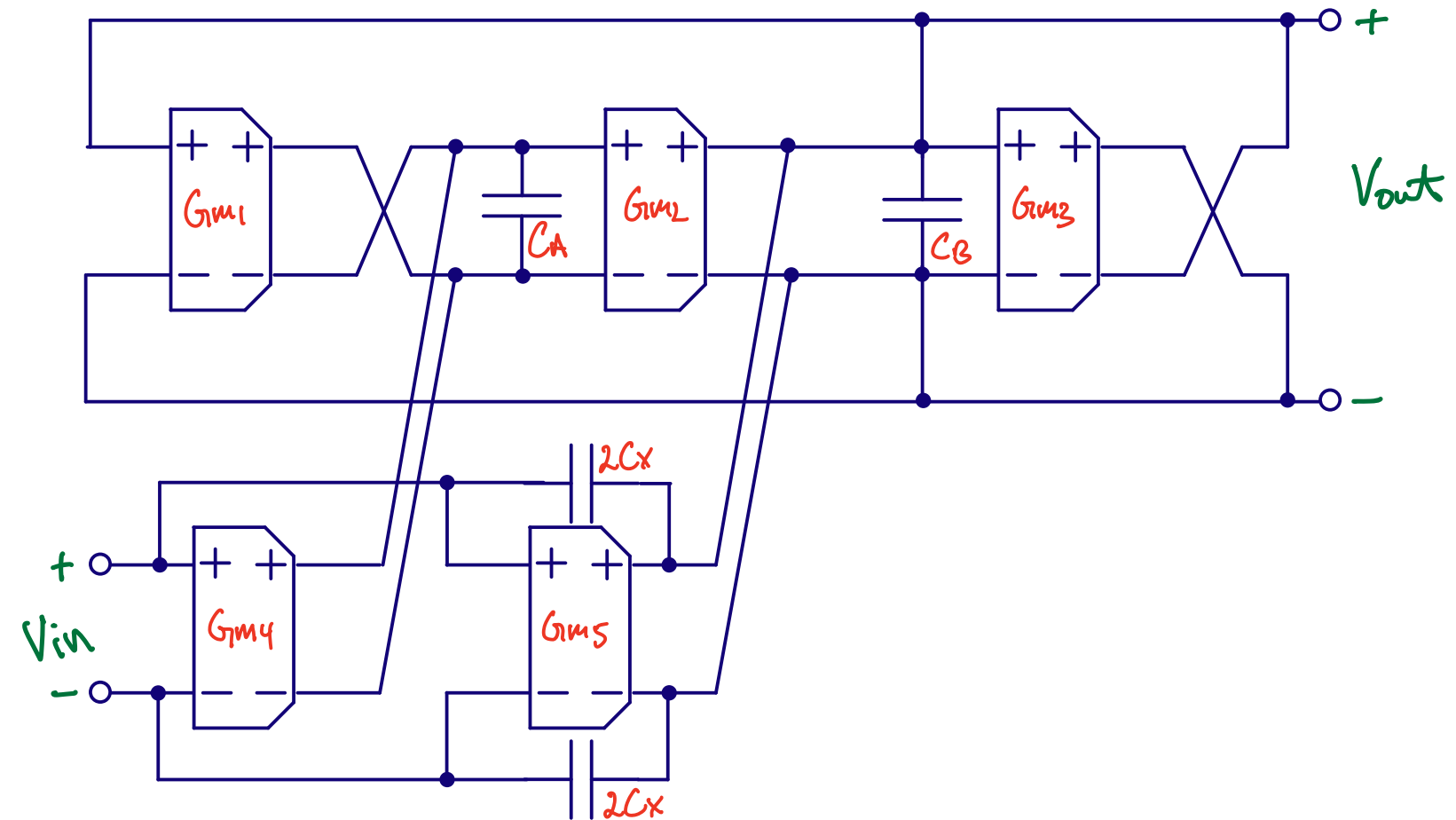
$$H(s) = \frac{k_1 s + k_0}{s + \omega_0}$$

$$H(s) = \frac{s \frac{C_x}{C_a + C_x} + \frac{G_{m1}}{C_a + C_x}}{s + \frac{G_{m2}}{C_a + C_x}}$$

Q: Try and calculate the transfer function

$$H(s) = \frac{k_2 s^2 + k_1 s + k_0}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2}$$

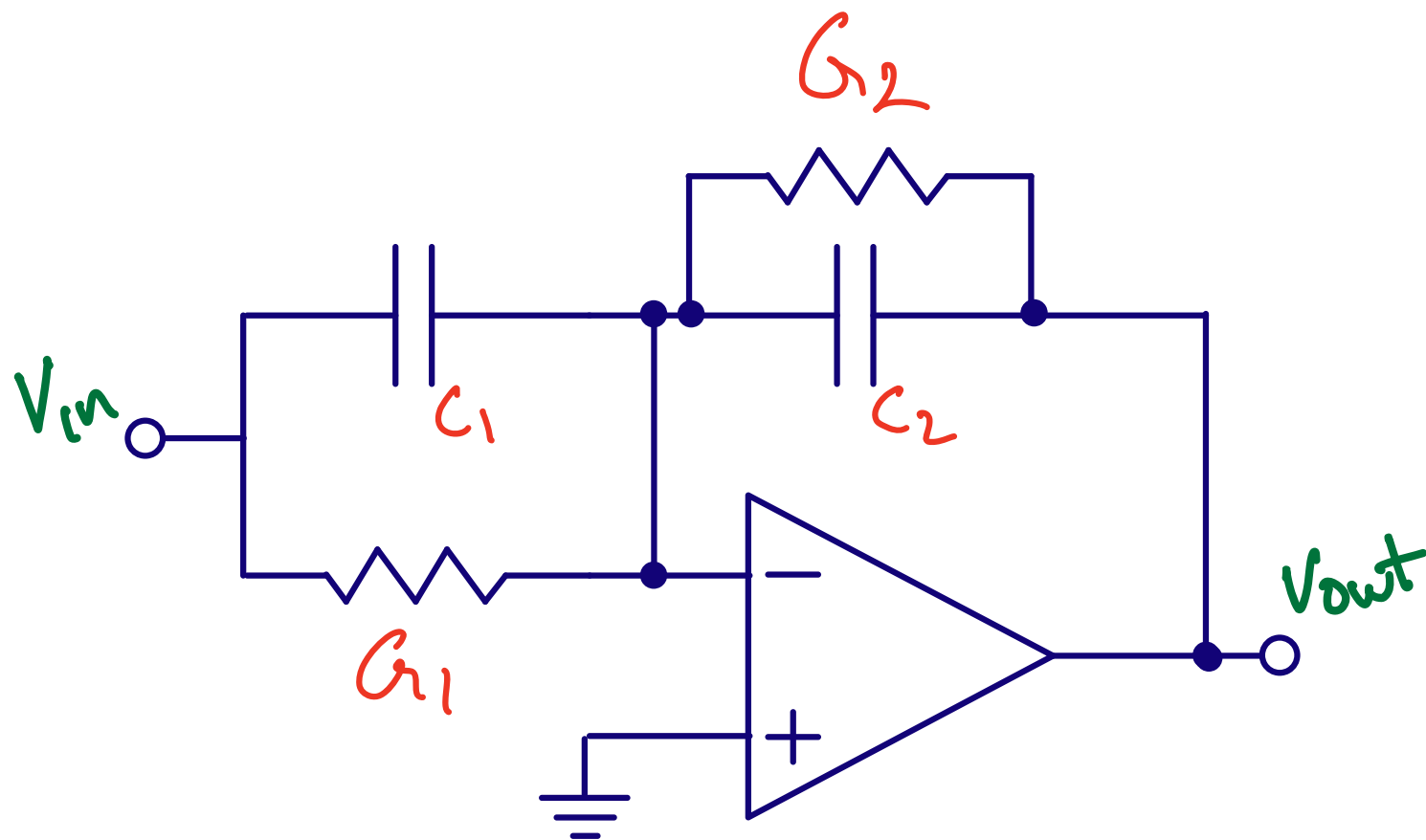
$$H(s) = \frac{s^2 \frac{C_X}{C_X + C_B} + s \frac{G_{m5}}{C_X + C_B} + \frac{G_{m2} G_{m4}}{C_A (C_X + C_B)}}{s^2 + s \frac{G_{m2}}{C_X + C_B} + \frac{G_{m1} G_{m2}}{C_A (C_X + C_B)}}$$



**Q: Try and figure out how we could make a
transconductor**

Active-RC

General purpose first order filter

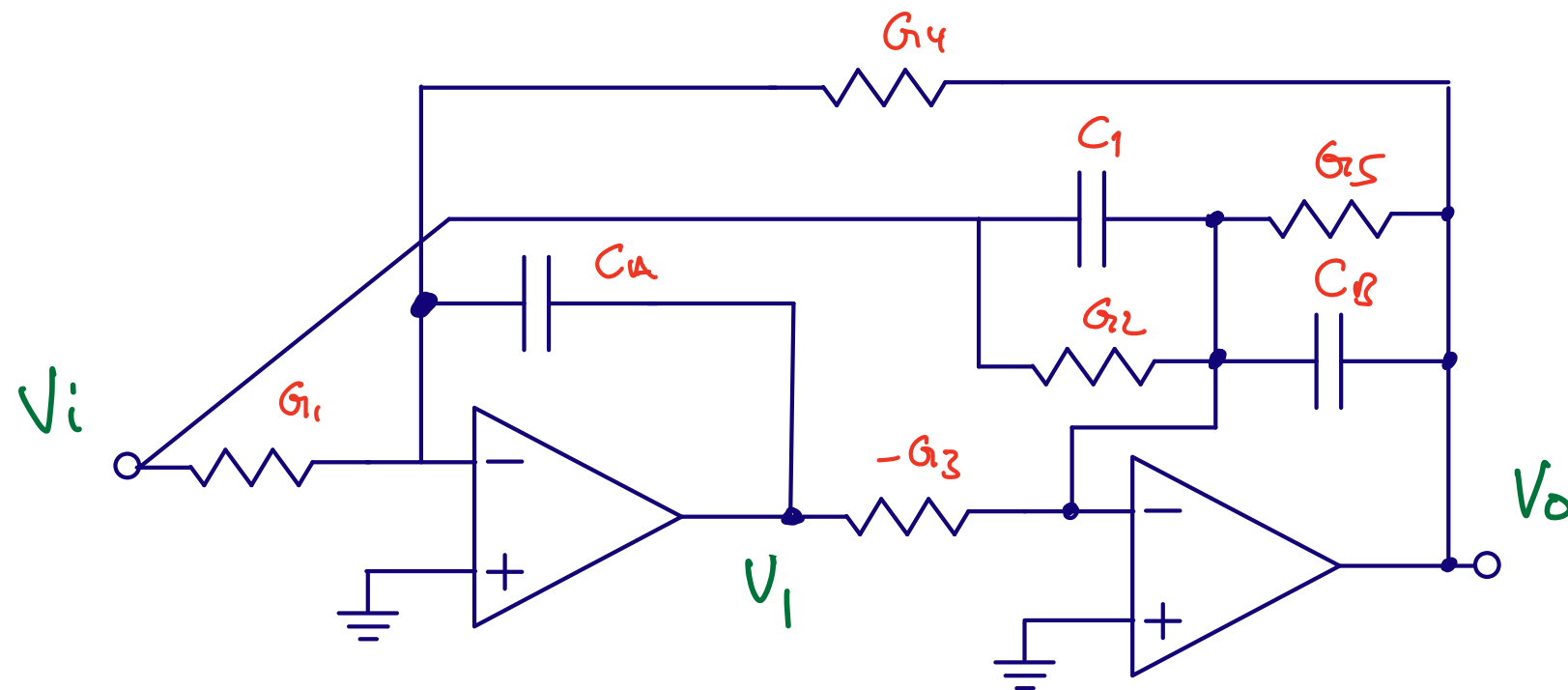


$$H(s) = \frac{k_1 s + k_0}{s + \omega_0}$$

$$H(s) = \frac{-\frac{C_1}{C_2} s - \frac{G_1}{C_2}}{s + \frac{G_2}{C_2}}$$

Q: Try and calculate the transfer function

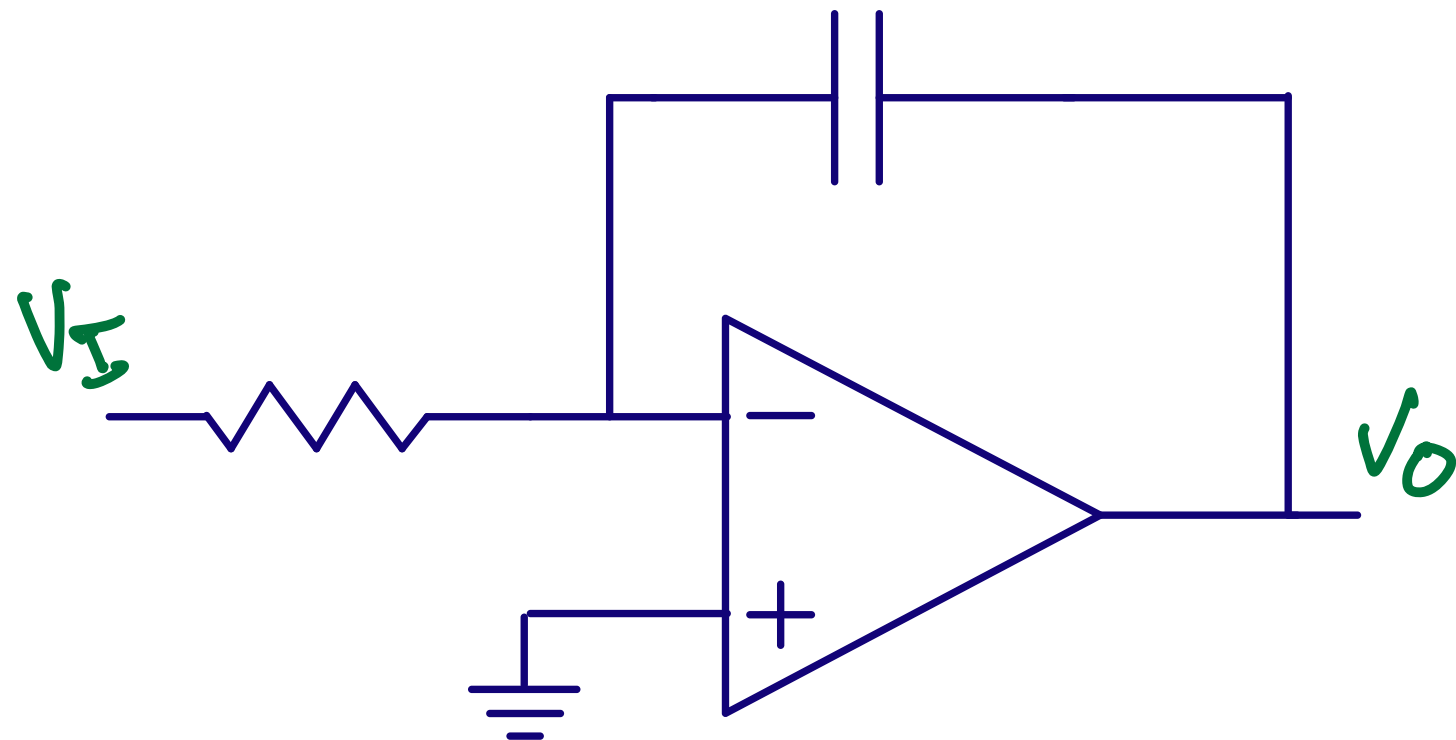
General purpose biquad



$$H(s) = \frac{k_2 s^2 + k_1 s + k_0}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2}$$

$$H(s) = \frac{\left[\frac{C_1}{C_B} s^2 + \frac{G_2}{C_B} s + \left(\frac{G_1 G_3}{C_A C_B} \right) \right]}{\left[s^2 + \frac{G_5}{C_B} s + \frac{G_3 G_4}{C_A C_B} \right]}$$

The OTA is not ideal

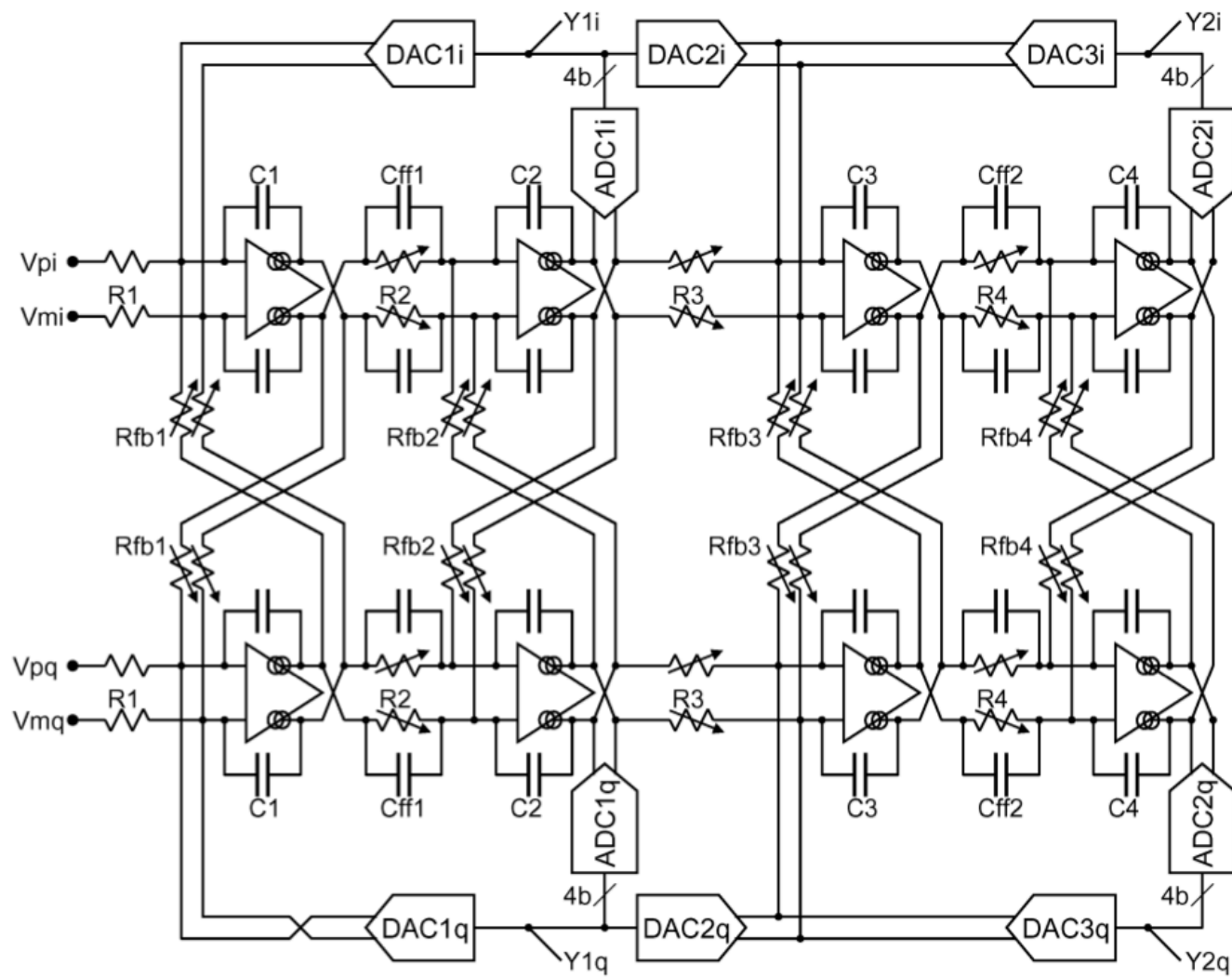


$$H(s) \approx \frac{A_0}{(1 + sA_0RC)(1 + \frac{s}{\omega_{ta}})}$$

where A_0 is the gain of the amplifier, and ω_{ta} is the unity-gain frequency.

Q: In what region does this equation match an ideal integrator $1/sRC$ response?

A 56 mW Continuous-Time Quadrature Cascaded Sigma-Delta Modulator With 77 dB DR in a Near Zero-IF 20 MHz Band



sigma-delta modulator design.

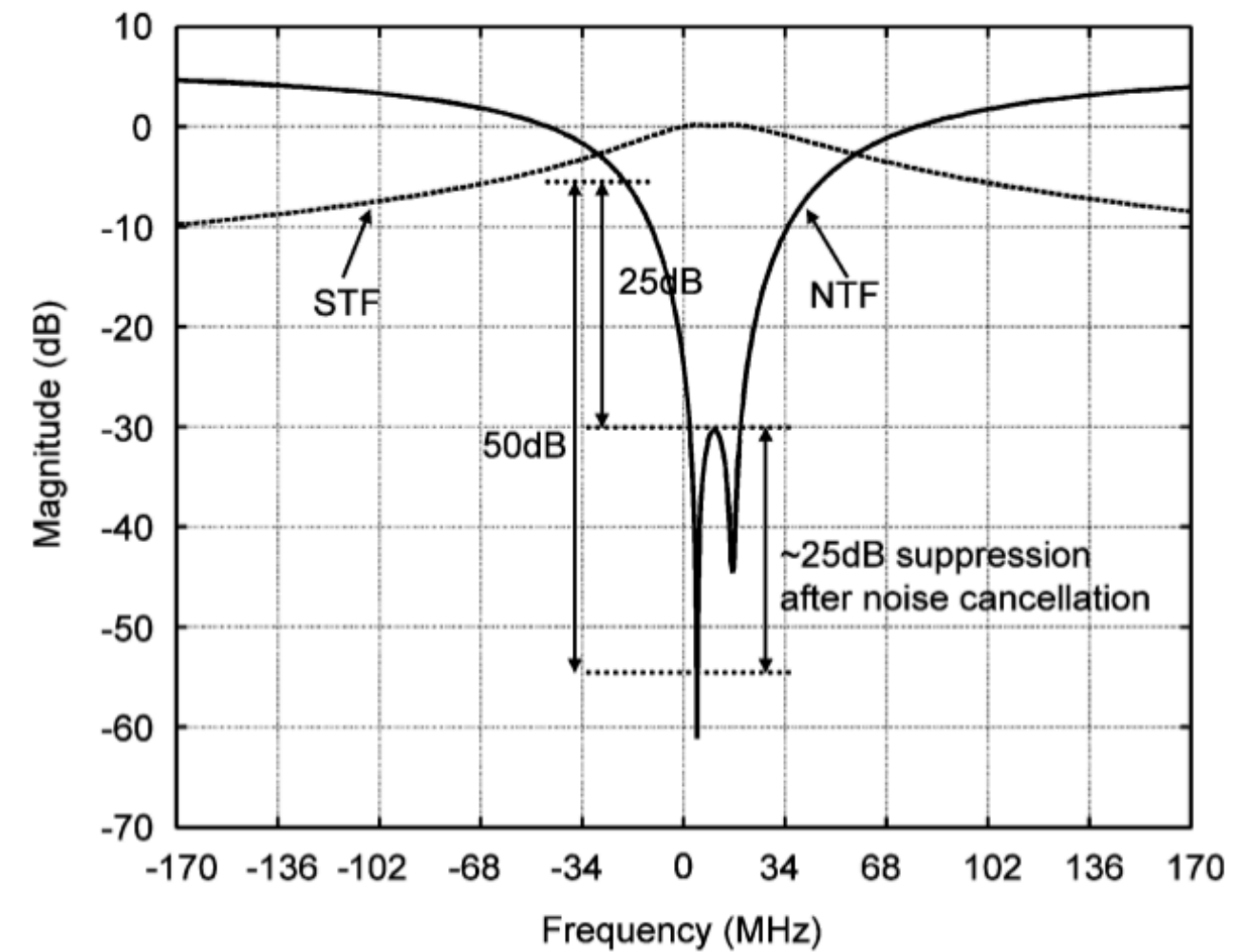


Fig. 9. NTF and STF of first stage.

Thanks!

